MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXXXVI. APRIL 9, 1926. No. 6

Dr. J. H. JEANS, PRESIDENT, in the Chair.

Arthur Tasker Burnell, Ribstone House, Sydney Road, Friern Barnet, London, N. 10;
James Chilton Dobbie, B.A., 229 Gloucester Terrace, Paddington, London, W.;
John William Harmer, B.A., Marlborough College, Wilts.;
Charles William Jeffries, Royal Observatory, Hong-Kong;
P. Samuels Lall, M.A., Professor, Forman College, Lahore, and Curator, Punjaub University Observatory, Lahore, India;
Nita Ermenilda Elvira Sharpe, Rosemead, Littlehampton, Sussex;
Duncan McLaren Young Sommerville, M.A., D.Sc., Professor of Mathematics, Victoria University College, Wellington, New Zealand; and
Hugh Chaloner Woods, B.Sc., Oakhurst, Blindley Heath, Surrey,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

William Ernest Allen, Lieut., R.N.R., 19 Manor Lane Terrace, London, S.E. 13 (proposed by L. J. Comrie);
William Harry Coombs, F.R.G.S., A.I.N.A., Royal Thames Yacht Club, Knightsbridge, London, S.W. (proposed by S. Fielder);
Alexander William Fenton, Thornwood and Broughton Nautical Schools, 4 and 5 Carlisle Avenue, London, E.C. 3 (proposed by E. Fawcett White);

Mohamed Reda Madwar, B.Sc., Ph.D., A.M.I.C.E., Helwan Observatory, Egypt (proposed by J. Storey);
Thomas Harvey Walmsley, F.R.G.S., c/o The Naraguta Tin Mines Ltd., Jos, Nigeria (proposed by D. Maclean); and
Hugh Montague Wood, 5 Carlisle Avenue, Fenchurch Street, London, E.C. 3 (proposed by S. Fielder).

Seventy-three presents were announced as having been received since the last meeting, including, amongst others :—

Astrographic Charts: Bordeaux Observatory, zones +14°, +12° (18 charts); Paris Observatory, zone +20° (1 chart); Toulouse Observatory, zones +5°, +7°, +9° (9 charts).
Astrographic Catalogue: San Fernando Observatory, Vol. vi, zone −7°.

Celestial Photographs.

The following have been added to the list of Celestial Photographs reproduced by the Society for sale to the Fellows:—

R.A.S.
Ref. No.
367. Halley's Comet, 1910 June 2 (Exp. 30 min.), 30-in. Refl., Helwan.
368. Field of Orion, (Exp. 2 hrs.), 10-in. lens, Franklin-Adams.

The Photometric Scale of the Leander McCormick Observatory.

By S. A. Mitchell and H. L. Alden.

After Nova Persei was placed under observation at the Leander McCormick Observatory for the purpose of determining the fluctuations in its brightness (described in the following pages) the star became of special interest to us. The chief observational work of the observatory is the determination of stellar parallaxes by photography with the 26-inch visual refractor, but during the hours near midnight the telescope is available for other lines of research.

About two hundred variables of long period, which at minimum become fainter than the thirteenth magnitude, are being followed, the observations being made primarily when the variables are faint. In addition to these observations by the Argelander method a plan was inaugurated several years ago of independently determining the magnitudes of the comparison stars in each region under observation. Not only have the magnitudes of the comparison stars adopted by Harvard been determined, but the sequences have been extended, when necessary, to stars fainter than the fifteenth magnitude.

For deriving the magnitudes there was available a wedge photo-

meter, which had been extensively used by both of us in the extended programme of determining the "photometric magnitudes of faint standard stars measured visually at Harvard, Yerkes, Lick, and McCormick Observatories." This work is described in the *Memoris of the American Academy of Arts and Sciences*, **14**, 213, 1923. It was there shown that the scales of magnitudes given by the photometers of the co-operating observatories did not agree. The greatest difference was found in the scale of Parkhurst at Yerkes on the one hand, and Mitchell and Alden at the McCormick Observatory on the other. On the average our scale was about 12 per cent. more extended than that of Parkhurst, with the result that in going from the brighter stars used as standards to the faintest stars of the sequence, Parkhurst's magnitudes were about half a magnitude brighter than ours.

This uncertainty of scale was of great concern to us. For the further investigation of this difference it was a matter of great regret to us that the mounting of the 26-inch telescope did not permit us to use the photometer on the polar sequence. We did not know of any visual magnitudes of high reliability that extended to the limit of our telescope, the sixteenth magnitude. Consequently, we were much pleased to have the magnitudes published by Seares * in the region of Nova Persei. Both photographic and photovisual magnitudes are given extending from 11·78 to 15·88.

The verification of the scale of the McCormick magnitudes was of interest to us, not only on account of the Rumford work on faint standard stars, but also as furnishing an indication of how closely our scale agreed with the international standard in our work on the redetermination and extension of the Harvard magnitudes in the sequences of long period variables. From our point of view the solution of our problem has given a very satisfactory answer, in that the McCormick visual scale agrees remarkably closely with the photovisual scale of Mount Wilson.

In the past ten years each of us has become gradually trained in the determination of visual magnitudes, and this has been accomplished not only by the continued use of the photometer but by the estimation of grades by the Argelander method. As a result of these continued observations the grade, or step, of each of us has become gradually standardised to such an extent that from the eighth magnitude (observed with the 5-inch finder) to the sixteenth magnitude (observed in the 26-inch), the grade of each is constant, and is closely equal to one-tenth of a magnitude. In other words, our grades plotted against magnitudes give a straight line.

In accordance with our usual custom when observing with the photometer we worked together, Alden bringing the star to be observed into the proper position with respect to the artificial star, and making his settings of the photometer wedge, Mitchell then following by making his observations. By the use of a recording device on the photometer it was unnecessary to read the scale at the time of observation, with the result that our observations were entirely independent. However, when making our observations visually by the Argelander method we worked separately.

* *Mt. Wilson Contributions*, 192 : *Ap. Jour.*, **52**, 183, 1920.

For deriving the magnitudes of the comparison stars in the field of Nova Persei each of us used the photometer on three nights, the faintest star measured being of magnitude 15·19. Each of us separately on three nights determined the grades between the successive stars in the sequence from the brightest star 10·63 to the faintest, the photovisual magnitude of which is 15·88.

In the following table the results are confined to those stars immediately surrounding Nova Persei which were used by Barnard * and for which Seares gives both the photographic and photovisual magnitudes (and consequently the colour-index). Both Alden and Mitchell separately and independently reduced the McCormick observations and derived the magnitudes, the values in the table below, however, being from the discussion of Mitchell. In column (1) are given the Mt. Wilson photovisual magnitudes, in column (2) the colour-index, in column (3)

Comparison of McCormick Magnitudes with Photovisual of Mt. Wilson.

Mt. W. Pv.	Colour-Index.	Photometer.	Grades.	Grades and Photometer.	Grades and Mt. W.	Grades +10·60.	Adopted.	(1)—(8).	Barnard.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
11·78	+0·18	11·54	11·5	11·73	11·78	11·75	11·64	+·14	..
12·32	+0·88	12·25	17·0	12·29	12·32	12·30	12·27	+·05	12·35
12·64	+0·73	12·52	20·3	12·63	12·65	12·63	12·58	+·06	12·75
13·16	+1·08	13·21	26·2	13·20	13·23	13·22	13·20	−·04	13·01
13·20	+0·87	13·14	24·9	13·07	13·08	13·09	13·10	+·10	13·19
13·47	+1·10	13·33	28·9	13·48	13·47	13·49	13·40	+·07	13·48
13·60	+0·84	13·50	29·3	13·54	13·56	13·53	13·52	+·08	13·60
13·67	+0·75	13·56	30·7	13·67	13·67	13·67	13·62	+·05	13·66
13·68	+0·82	13·74	30·7	13·67	13·67	13·67	13·70	−·02	13·70
13·82	+0·89	13·90	32·0	13·81	13·81	13·80	13·86	−·04	13·62
13·94	+0·57	13·94	33·3	13·93	13·93	13·93	13·94	·00	13·94
13·94	+1·67	14·09	33·6	13·96	13·96	13·96	14·02	−·08	14·07
14·20	+1·41	14·56	36·8	14·28	14·27	14·28	14·42	−·22	14·30
14·22	+1·13	14·27	35·6	14·16	14·15	14·16	14·22	·00	14·24
14·38	+0·82	14·50	37·8	14·37	14·37	14·38	14·44	−·06	14·39
14·53	+1·04	14·44	39·8	14·59	14·56	14·60	14·52	+·01	14·51
14·74	+0·98	14·65	41·8	14·79	14·75	14·78	14·72	+·02	14·43
15·01	+0·61	15·12	44·8	15·09	15·05	15·08	15·10	−·09	..
15·19	+1·25	15·17	46·6	15·27	15·23	15·26	15·22	−·03	15·30
15·88	+0·98	..	52·6	15·87	15·86	15·86	15·86	+·02	..

* *Monthly Notices*, **80**, 710, 1920.

the McCormick photometer, the mean of the observations of Mitchell and Alden, in (4) the grades determined from the means of our observations. In column (5) the grades are plotted against the photometer values in column (3), and in (6) the grades are plotted against the Mt. Wilson magnitudes. In order to exhibit how closely our grade approximates to one-tenth of a magnitude, the grades when divided by 10 are added to 10·60 as zero point, and the results are given in column (7). In the following column (8) a mean is taken of the results in columns (3) and (5), and these are the adopted values of the McCormick magnitudes. The following column gives the difference between Mt. Wilson and McCormick, *i.e.* (1)–(8). The final column gives Barnard's values (*loc. cit.*) with a correction of +0·80 magnitudes for zero-point.

From the differences found in column (9) a probable error ±0·06 magnitudes is derived, and if we give equal weight to the Mt. Wilson and McCormick magnitudes then there results a probable error of a single magnitude for either observatory of ±0·04 magnitudes.

However, the differences in column (9) include not only all observational errors of Mt. Wilson and McCormick but also all systematic differences depending on scale and on the colour of the star. In order to ascertain the relations between the scales of Mt. Wilson and McCormick and the dependence of the magnitude on colour, equations were formed for the magnitudes of each star as follows :

$$\text{McC.}=\text{Mt.W.}+c+a(\text{Mt.W}-14^{\text{m}}\cdot0)+b(\text{colour-index})$$

where c is the correction for zero-point, a the scale factor, and b the dependence on colour. Giving the last star half weight (since it was not observed by photometer), and solving by least squares, then there results :

$$\text{McC. (photometer and visual)}=\text{Mt. W.}+0^{\text{m}}\cdot014+0\cdot031\ (\text{Mt. W.}-14^{\text{m}}\cdot0)+0^{\text{m}}\cdot108\ (\text{C.I.}-1\cdot00).$$

Using the McCormick photometer only and making similar reductions then we derive :

$$\text{McC. (photometer only)}=\text{Mt. W.}+0^{\text{m}}\cdot020+0\cdot044\ (\text{Mt. W.}-14^{\text{m}}\cdot0)+0^{\text{m}}\cdot144\ (\text{C.I.}-1\cdot00).$$

From the discussion of the results independently carried out by Alden, and referred to above, the values for the combined photometer and visual magnitudes give the equation :

$$\text{McC.}=\text{Mt. W.}+0^{\text{m}}\cdot025+0^{\text{m}}\cdot025\ (\text{Mt. W.}-14^{\text{m}}\cdot0)+0^{\text{m}}\cdot160\ (\text{C.I.}-1\cdot00).$$

The only difference in the reductions by the two of us arises from the manner in which the grade-estimates were plotted against the photometer values. Since each discussion is entitled to equal weight the connection between the McCormick magnitudes and the Mt. Wilson photovisual results can best be represented by the mean of the two equations, and this mean gives :

$$\text{McC.}=\text{Mt. W.}+0^{\text{m}}\cdot020+0^{\text{m}}\cdot028\ (\text{Mt. W.}-14^{\text{m}}\cdot0)+0^{\text{m}}\cdot134\ (\text{C.I.}-1\cdot00).$$

From the discussion of the observations made by us with the wedge photometer of stars in the Pleiades * when compared with the Mt. Wilson photovisual magnitudes there results the correction :

McC. (photometer)=Mt. W.$-0^{m}\cdot333-0^{m}\cdot006$ (Mt. W.$-13^{m}\cdot0$)
$+0^{m}\cdot244$ (C.I.$-1\cdot00$).

If we take the mean of the corrections to the scale of the McCormick magnitudes from the measures on the Pleiades and Nova Persei we see that the McCormick scale is 1 per cent. less extended than the photovisual scale of magnitudes of the Mt. Wilson Observatory. This latter scale is in substantial accord with the international standard. As one would expect, the visual magnitudes differ from the photovisual magnitudes (determined by photography by means of yellow light) by showing a dependence on colour amounting to one-fifth of a magnitude for a change in the colour-index of one unit. This colour value is caused partly by the different manner in which the eye sees colour from that of the photographic plate with yellow light, and partly by differences resulting from the telescopes employed.

The photovisual magnitudes of the two brightest stars of the sequence are 10·63 and 10·75, while the McCormick values are 10·47 and 11·08. On account of the large difference in magnitude for the second star it appears probable that this star is variable.

Leander McCormick Observatory,
University of Virginia :
1926 *March* 1.

Observations on Nova Persei (1901) *made with the 26-inch Refractor of the Leander McCormick Observatory.* By S. A. Mitchell and H. L. Alden.

While visiting at Yerkes Observatory in the summer of 1922, a guest in the home of Professor Barnard, one of us discussed with him the observations which he had been making on Nova Persei, and he requested that this interesting star be placed under observation at the Leander McCormick Observatory. The idea was that by this means observations could be made on the same nights, and thus it would be possible to separate the fluctuations in brightness of the Nova from accidental errors of observation.

Unfortunately the death of Professor Barnard, six months later, put a stop to the successful completion of this plan. By this co-operative scheme the Nova would have been observed frequently by two refractors of much greater aperture than are ordinarily employed in the observation of variables. The plan was further interfered with by the departure of one of us to South Africa.

The McCormick observations were reduced by the photovisual magnitudes of Seares (given in the preceding pages), and consequently these magnitudes are directly comparable with Barnard's, Steavenson's,

* *Astronomical Journal,* **36**, 73, 1925.

and Peek's observations. Our results confirm the frequent and sudden changes in brightness of the Nova shown by other series of observations. We give, in the table below, the number of stars used in making the comparisons with the Nova.

Date.	J.D.	Magn.	No. of Comps.	Obs.
1922 Nov. 19	2423378·67	13·47	6	A
22	381·80	13·32	9	A
23	382·59	13·04	4	O
25	384·59	13·04	2	O
28	387·75	13·49	8	A
29	388·59	13·44	3	O
30	389·63	13·44	3	M
Dec. 1	390·55	13·66	6	A
5	394·75	13·52	4	A
12	401·78	12·44	4	A
17	406·71	12·90	4	M
19	408·83	12·91	10	A
21	410·58	13·63	7	O
21	410·59	13·65	8	A
23	412·50	13·14	7	A
23	412·51	13·15	4	O
26	415·57	13·00	6	A
29	418·74	13·01	6	A
1923 Jan. 1	421·51	12·71	4	A
5	425·78	13·11	3	A
8	428·50	12·93	4	B
10	430·51	12·88	5	A
12	432·51	12·88	3	O
13	433·61	12·78	4	B
15	435·72	12·75	2	B
16	436·50	12·74	3	O
19	439·50	12·84	3	O
22	442·50	13·16	1	B
24	444·62	13·58	5	M
25	445·65	13·33	4	A
28	448·62	13·35	4	M
Feb. 7	458·67	13·73	5	A
8	459·63	13·65	3	A
14	465·55	13·67	6	A
18	469·63	13·76	4	M
21	472·60	13·77	6	A
Mar. 4	483·50	13·80	3	M
8	487·62	13·76	3	M
1923 Mar. 28	2423507·54	13·64	2	A
Apr. 5	515·58	13·62	6	A
Aug. 15	647·90	13·04	6	A
Sept. 9	672·90	12·82	6	A
16	679·79	12·84	7	A
Oct. 12	705·70	12·43	2	M
31	724·69	12·50	2	M
Nov. 1	725·60	12·43	2	A
10	734·74	12·34	3	A
14	738·62	12·83	4	M
20	744·74	12·78	4	A
Dec. 3	757·75	13·65	3	A
23	777·50	13·75	8	A
1924 Jan. 3	788·67	13·66	6	A
27	812·69	13·82	6	M
Feb. 21	837·51	13·65	9	A
Mar. 2	847·62	13·90	4	M
Oct. 3	2424062·75	13·77	5	A
4	063·76	13·35	8	M
5	064·73	13·19	8	M
6	065·58	13·08	2	A
8	067·73	13·11	5	M
9	068·71	13·46	7	A
9	068·71	13·60	7	M
10	069·79	13·62	7	A
11	070·75	13·70	5	M
13	072·69	12·93	3	M
14	073·75	13·10	4	A
16	075·69	13·02	5	A
17	076·70	13·70	5	M
18	077·75	13·63	7	M
20	079·67	13·18	2	A
22	081·71	13·60	7	M
23	082·71	13·19	4	M
24	083·53	13·38	4	M
24	083·76	13·40	7	A
31	090·55	13·18	6	M
Nov. 1	091·60	13·69	5	A

Date.	J.D.	Magn.	No. of Comps.	Obs.
1924 Nov. 2	2424092·73	13·20	6	M
3	093·54	13·08	4	A
3	093·68	13·18	6	M
5	095·55	13·68	5	M
13	103·50	13·57	7	A
14	104·54	13·58	4	M
15	105·76	13·76	4	M
16	106·58	13·46	5	M
17	107·51	13·44	6	A
19	109·61	13·65	10	A
20	110·58	13·55	6	M
24	114·65	13·65	5	M
26	116·66	13·49	7	M
29	119·54	13·79	6	A
29	119·70	13·60	7	M
Dec. 1	121·62	13·67	4	A
3	123·57	13·48	8	A
9	129·55	13·76	3	M
17	137·53	13·52	7	A
20	140·72	13·61	3	V
21	141·71	13·76	6	M
1924 Dec. 22	2424142·52	13·49	7	A
26	146·62	13·69	5	M
28	148·56	13·25	4	A
1925 Jan. 5	156·56	13·56	7	A
8	159·55	13·63	4	A
20	171·52	13·03	3	A
30	181·67	13·53	7	M
31	182·60	13·50	6	A
Feb. 26	208·60	13·67	3	M
Mar. 2	212·49	13·78	6	A
15	225·60	13·76	10	M
20	230·62	13·58	8	A
23	233·60	13·67	5	A
Nov. 8	463·75	13·44	5	M
13	468·60	13·58	5	M
Dec. 8	493·70	13·44	5	M
1926 Jan. 10	526·72	13·66	5	M
12	528·67	13·61	5	M
Feb. 14	561·59	13·76	6	M
Mar. 4	579·58	13·85	6	M
7	582·58	13·76	6	M

Observers.

A=Harold L. Alden.
B=J. W. Blincoe.
M=S. A. Mitchell.
O=Chas. P. Olivier.
V=A. Vyssotsky.

Leander McCormick Observatory,
University of Virginia,
1926 *March* 10.

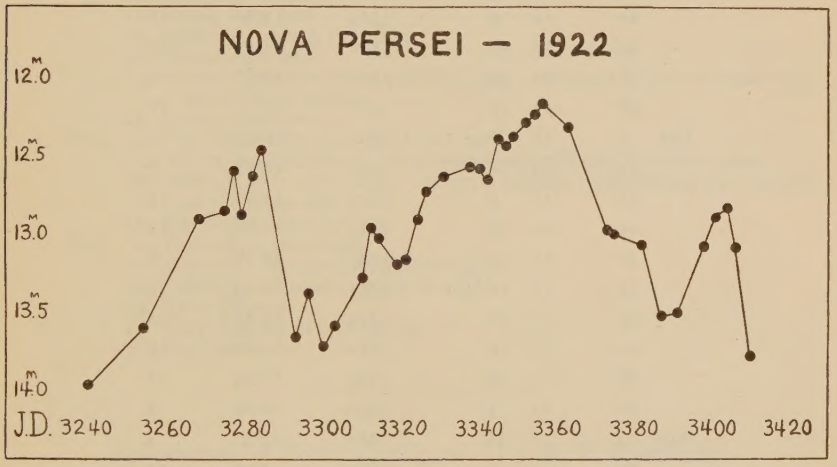
Visual Observations of the Brightness of Nova Persei No. 2 (*Anderson* 1910). By the late E. E. Barnard.

(*Communicated by Professor E. B. Frost.*)

[*Note by Professor Frost.*—The following observations, made with the 40-inch refractor of the Yerkes Observatory, are a continuation of those printed in *Monthly Notices*, **80**, 710–722, and **82**, 457–461. The magnitudes were derived from comparisons with stars of a list of nineteen for which magnitudes had been determined photovisually by Professor F. H. Seares of the Mount Wilson Observatory. Their magnitudes are given in the papers referred to above. Although the observations of the

Nova are given to the hundredth of a magnitude, only the tenth of a magnitude may be taken as significant.

These observations had been reduced before Professor Barnard's death, on 1923 February 6, but their publication has been delayed so that they might appear in the same number of the *Monthly Notices* with a series of observations of the same star by Professor S. A. Mitchell and Dr. H. L. Alden, made at the M'Cormick Observatory of the University of Virginia.



A plat has been drawn by Miss Calvert for these observations, similar to that used in the last preceding paper (**82**, 457).

The nineteen estimates of magnitude from thirteen comparison stars on the night of 1922 December 21 were Professor Barnard's last visual observations with the 40-inch telescope.]

Visual Magnitude Observations of Nova Persei.

Date.	G.S.T.*	J.D.	Mag.	No. of Comps.
1922	h m			
July 4	15 0	2423240	13·98	4
18	14 30	3254	13·61	12
Aug. 1	15 45	3268	12·91	10
8	15 30	3275	12·86	10
10	13 45	3277	12·60	9
12	13 20	3279	12·88	9
15	13 50	3282	12·63	8
17	13 20	3284	12·47	8
26	13 10	3293	13·67	13
29	13 30	3296	13·39	12

* The times are 6^h 0^m slow on Greenwich mean time.

Visual Magnitude Observations of Nova Persei—continued.

Date.	G.S.T.*	J.D.	Mag.	No. of Comps.
1922	h m			
Sept. 2	12 15	2423300	13·73	14
5	13 50	3303	13·60	12
12	13 15	3310	13·28	13
14	13 20	3312	12·96	14
16	13 0	3314	13·03	12
21	12 0	3319	13·20	15
23	10 30	3321	13·17	10
26	12 40	3324	12·91	10
28	12 45	3326	12·74	11
Oct. 3	17 0	3331	12·63	7
11	13 0	3338	12·57	6
12	12 5	3340	12·58	15
14	12 55	3342	12·66	15
17	10 35	3345	12·38	9
19	11 10	3347	12·43	10
21	10 50	3349	12·37	12
24	12 10	3352	12·28	11
26	12 20	3354	12·24	11
28	12 5	3356	12·16	9
Nov. 4	8 45	3363	12·31	4
14	9 0	3373	12·98	12
16	11 10	3375	13·00	11
23	11 10	3382	13·07	11
28	8 5	3387	13·53	13
Dec. 2	8 0	3391	13·51	13
9	9 40	3398	13·08	11
12	9 20	3401	12·88	11
15	10 35	3404	12·84	10
17	9 20	3406	13·08	13
21	10 25	3410	13·79	19

Notes.

1922
July 4. Very low; dawn and bad seeing.
18. Somewhat hazy, though stellar.
Aug. 1. In dawn.
8. Moonlight and bad seeing.
10. Difficult in moonlight and bad seeing.
15. In moonlight. Does not differ in appearance from other stars.
17. Very bad seeing.
26. Faint to-night. The nebulosity is noticeable and affects comparisons. Its colour is pale white.

* The times are $6^h\ 0^m$ slow on Greenwich mean time.

1922

Sept. 2. Fainter to-night.
5. Moonlight and bad seeing.
14. Through breaks in clouds. Hazy, but white; no difference in focus from other stars.
23. Through clouds. Seeing very bad.
28. It seems brighter to-night. Rather difficult; seeing very bad.

Oct. 3. It is bright. Uncertain; through clouds.
11. Through clouds.
12. Very bad seeing.
14. Difficult; clouds.
17. Seeing very bad.
19. Difficult; seeing very bad.
21. Seeing very bad.
24. It seems brighter than usual to-night. Seeing too bad to make anything of the nebula.
26. Seeing very bad.

Nov. 4. Very difficult in moonlight and haze.
14. White, but ill-defined; very hazy compared with the other stars.
16. Difficult from bad seeing. It has a nebulous look, not as well defined as the other stars.
28. Moonlight and poor sky.

Dec. 2. Difficult in moonlight.
9. Difficult. It seems bright to-night.
12. Difficult. It seems bright to-night.
15. Very difficult.
17. Very difficult in bad seeing.
21. Very faint.

Observations of Novæ, 1925–1926. By W. H. Steavenson.

The photometric observations here recorded are in continuation of those made in former years and already communicated to the Society. Most of them were made with the 6-inch Wray refractor, presented to the writer by Mr. J. E. Drower; but for some of the fainter Novæ the 28-inch refractor of the Royal Observatory was used, by kind permission of the Astronomer Royal. In the very few cases where observations were made after midnight the date given is that of the day just completed.

Nova Cassiopeiæ (1572).

The field of this Nova was examined on 32 nights between 1925 June 18 and December 6. Special attention was paid to the two twelfth-magnitude stars near to the reputed position of the Nova, but no variation was noted in the light of these or of any other stars in the field.

Nova Vulpeculæ (1670).

The identity of this Nova is still a matter of doubt. The late Professor Barnard gave a map of the field in *A.N.*, 4753, 1, and most of the faint stars recorded thereon have been identified with the 6-inch telescope. The field was examined on 7 nights between 1925 July 27 and September 14, but no changes of brightness were observable in any of the stars.

Nova Ophiuchi (1848).

Observed on 13 nights between 1925 May 12 and September 8. Very little change in brightness was noted, though in past years the variability of this Nova has often been quite pronounced.

Date.	Mag.	Date.	Mag.
1925 May 12	12·4	1925 July 7	12·45±
25	12·5	10	12·5±
28	12·5	27	12·4
June 9	12·45	28	12·35
10	12·4	Sept. 6	12·55
13	12·45	8	12·6
18	12·5±		

Nova (T) *Coronæ* (1866).

Observed on 36 nights between 1925 July 28 and 1926 April 5. This star has now been practically quiescent for a great number of years. It still seems, however, to be very slightly unsteady in its light, and, although the amount of its variability is but little greater than the probable error of the observations, the following estimates are given for what they may be worth.

Date.	Mag.	Date.	Mag.
1925 July 28	9·86	1925 Sept. 18	10·21
30	9·91	19	10·16
Aug. 1	9·91	21	10·16
2	9·86	22	10·11
5	9·86	23	10·16
7	9·81	24	10·26
10	9·96	26	10·11
15	9·96	27	10·16
16	9·96	Oct. 5	10·21
22	9·96	24	10·06
28	9·96	30	10·11
29	9·91	Nov. 2	10·06
31	9·96	1926 Jan. 27	10·06
Sept. 6	10·01	Mar. 13	10·01
8	10·01	15	9·94
11	9·96	31	9·96
12	10·16	Apr. 2	10·00
14	10·03	5	9·96

Nova Cygni (1876).

Observed on 9 nights between 1925 June 10 and September 8. The observations show no changes of brightness greater than the probable error of the estimates. On all but two occasions the magnitude was considered to be 14·9. On June 10 and July 12 the estimates were, respectively, 14·7 and 14·6. The Nova is always very near the limit of visibility with the 6-inch telescope.

Nova Aurigæ (1892).

Observed on 2 nights (1926 January 9 and 20) with the 28-inch refractor. The Nova was easily identified from a chart published by Burnham in *M.N.*, **52**, 433. On both occasions it was noted as being about a fifth of a magnitude fainter than the small star which immediately follows it. This star (Burnham's " B ") is between the 14th and 15th magnitude. The image of the Nova appeared perfectly stellar and no nebulosity was seen in its vicinity.

Nova Persei (1901).

Observed on 70 nights between 1925 June 28 and 1926 April 17. The variations of brightness observed during this period were somewhat more marked than those of 1924–25, though less so than those of some previous years. During the first half of the period the changes in light were of comparatively small amplitude, scarcely exceeding half a magnitude. Later the range became greater and the period of the variations much longer. What little evidence there is of a period previous to mid-November seems to point to one of about 8 or 10 days ; but from November 12 onwards there are three well-marked oscillations of period about 30 days and a probable amplitude of nearly a magnitude. After this the Nova became very faint and showed but slight variations of brightness. The extreme values for the magnitude during the entire period covered by the observations were 12·43 and 13·93, a range of exactly 1·5 magnitudes. The mean brightness of the Nova was 13·3, as compared with 13·5 in 1924–25. The continuity of the observations in 1925–26 has suffered more than usually from the effects of bad weather. In the following list of estimates the letter G denotes the 28-inch refractor of the Royal Observatory, Greenwich.

Date.	Mag.	Date.	Mag.	Date.	Mag.
1925 June 28	12·95	1925 Aug. 31	13·28	1925 Sept. 12	13·67
July 7	12·91	Sept. 1	13·28	13	13·42
10	13·06	3	13·62	14	13·47
27	13·23	4	13·57	19	13·42
28	13·18	6	13·57	21	13·72
30	13·67	7	13·67	23	13·47
Aug. 27	13·32	8	13·42	24	13·57
28	13·42	9	13·37	26	13·42
29	13·33	11	13·67	Oct. 5	13·37

Date.	Mag.	Date.	Mag.	Date.	Mag.
1925 Oct. 8	13·82	1925 Dec. 13	13·37	1926 Jan. 27	12·74
14	13·28	24	12·64 (G)	29	12·43
23	13·23	31	13·08	Feb. 4	12·64
24	13·28	1926 Jan. 2	13·18 (G)	15	13·57
30	13·33	3	13·32	28	13·72 (G)
Nov. 3	13·18	4	13·18 (G)	Mar. 3	13·77
4	13·18	6	13·08	4	13·82
11	13·78	7	13·18	9	13·88
12	13·52	9	13·13 (G)	10	13·82
16	12·94	10	13·03	15	13·72
17	13·18	12	13·42	20	13·93 (G)
Dec. 6	12·94 (G)	19	12·53	21	13·88 (G)
7	12·96	20	12·64 (G)	Apr. 15	13·87
8	13·23	25	12·53	17	13·57
9	13·42				

The wisp of nebulosity attached to the Nova remains unchanged and was observed several times with the Greenwich telescope.

Nova Geminorum (1903).

The field of this Nova was examined with the 28-inch refractor on 1926 January 9. The comparison stars given by Barnard in *M.N.*, **66**, 355, were identified, but no sign of the Nova was visible. It was concluded that it was fainter than magnitude 16. No nebulosity could be detected at or near the place formerly occupied by the Nova.

Nova Lacertæ (1910).

Observed on 6 nights between 1925 May 28 and July 28. On every occasion the magnitude was estimated at 14·1. There has been no certain change in the light of this Nova for many years, and the magnitude is practically the same as before its outburst in 1910.

Nova Geminorum (1912).

Observed with the 28-inch refractor on 2 nights, 1926 January 9 and 20. This Nova is now of about the 14th magnitude, being almost as faint as before its outburst in 1912. It is difficult to observe with low powers or with small apertures owing to its proximity to a star of about the 12th or 13th magnitude. The distance between the two is about 10″ or 12″. Both are shown on a chart published by the Royal Observatory, Greenwich, in *M.N.*, **72**, 516. The Nova is actually the more southerly of the two stars, though the circle on the chart referred to has been drawn with the bright companion as its centre. The Nova, on 1926 January 9, was estimated to be one-tenth of a magnitude fainter

than a small star two minutes of arc distant from it in a north preceding direction. This star is shown on the Greenwich diagram of the field. It is hoped later to make estimates of the magnitude of this and other suitable comparison stars.

The image of the Nova appears to be perfectly stellar in character, but it is involved in a faint nebulosity about 8″ in diameter. This nebulosity surrounds the Nova symmetrically and fades away diffusely in all directions. It is only possible to see it clearly by hiding the bright companion of the Nova behind the field-bar of the micrometer.

Nova Aquilæ (1918).

Observed on 41 nights between 1925 May 12 and November 17. All the estimates of magnitude range between 10·4 and 10·6, and it is perhaps doubtful whether any real variability was observed. The mean brightness is now almost exactly what it was before the outburst in 1918, and there has been little change in the past two or three years.

Nova Lyræ (1919).

Observed on 4 nights between 1925 July 26 and September 14. The Nova, which is only just visible in the 6-inch telescope, appears to be of magnitude 14·9±. No changes of brightness were observed.

Nova Cygni (1920).

Observed on 17 nights between 1925 May 25 and September 24. This Nova, which was fainter than the 15th magnitude before its outburst, has been fading steadily during the past five years. The fading has, however, been very slow since 1924, and in 1925 seems actually to have ceased for the time being. It seems probable, however, judging from the behaviour of other Novæ, that it will be continued again before long. All the estimates of magnitude made in 1925 lie between 13·13 and 13·28. There is no real evidence of variability, and the light may be said to have been sensibly constant.

Nova Aquilæ (1925).

This Nova was discovered by Dr. Max Wolf on a plate taken at Heidelberg on 1925 September 14. It was then of about the 9th magnitude, but nothing so bright as magnitude 16·5 was to be found in its place on plates taken in previous years. At the time of its discovery the Nova would seem not quite to have reached its maximum, which appears to have occurred during the last week of October. During the remainder of the year the changes were but slight, the maximum being unduly protracted. However, the observation of 1926 March 10 appears to show that the decline had then commenced, though the Nova was even then only about $0^{m}.6$ fainter than at the end of October. In the slowness of its light changes it seems to have something in common

with Nova Pictoris. It was observed on 24 nights between 1925 September 27 and 1926 March 10. There was nothing in its appearance to distinguish it from other stars of similar brightness. The colour appeared to be yellowish.

Date.	Mag.	Date.	Mag.
1925 Sept. 27	9·0	1925 Nov. 5	8·8
Oct. 5	9·0	11	8·85
10	8·92	12	8·8
11	8·93	17	8·7
24	8·8	19	8·8
25	8·6 (G)	27	8·8
27	8·8	Dec. 5	8·8
30	8·8	7	8·9
31	8·7	9	8·9
Nov. 2	8·82	15	8·8
3	8·65	23	8·6
4	8·8	1926 Mar. 10	9·3

Observations of Novæ, 1925–26. By B. M. Peek.

Photometric observations of Nova Ophiuchi (1848), Nova Cygni (1876), and Nova Persei (1901) have been continued in 1925–26, mainly with the 12¼-inch reflector. Owing, however, to the dismantling and moving of the observatory from Boscombe to Herne it was possible to continue only those of Nova Persei beyond the end of the year, as the 12¼-inch was out of commission for some time.

The individual observations are tabulated below.

Nova Ophiuchi, 1848.

J.D. [2424]	Mag.	J.D. [2424]	Mag.	J.D. [2424]	Mag.
256	12·45	319	12·5	397	<12·3
264	12·65	322	12·55	400	12·55
292	12·4	331	12·35±	401	12·55
296	12·4	342	12·4	406	12·55
301	12·4	343	12·4	407	12·55
302	12·4	346	12·45	408	12·55±
310	12·45±	351	12·4	417	12·5
311	12·4 ±	369	12·3 ±	418	12·5 ±
313	12·5 ±	370	12·45±	420	12·45±
316	12·45				

The comparison stars are those given by Barnard in *M.N.*, **81**, 167.

Nova Cygni, 1876.

J.D. [2424]	Mag.	J.D. [2424]	Mag.	J.D. [2424]	Mag.
264	<14·44	351	14·91	417	14·91
296	<14·71	400	14·91	429	14·86
316	14·76	401	14·96	432	14·86
322	14·61	402	14·94	438	14·84
342	<14·8	404	14·91	441	14·91
343	14·83	406	14·88	459	14·86
345	<14·85	407	14·91±	472	14·67±
346	14·9 ±	408	14·9 ±		

The magnitudes of the comparison stars are those given by Seares as used by Barnard and Steavenson.

Nova Persei, 1901.

J.D. [2424]	Mag.	J.D. [2424]	Mag.	J.D. [2424]	Mag.
333·56	13·02	420·37	13·36	465·44	13·77
344·56	<13·1	421·41	13·57	466·38	13·74
351·49	13·06	427·33	<13·2	467·34	13·55
365·48	13·4 ±	429·31	13·40	468·48	13·60
369·48	13·47±	430·33	13·37	471·34	13·30
370·48	<13·18	432·35	13·70	472·35	13·54
390·48	13·33	433·43	13·55	473·44	13·21
391·46	<13·18	434·41	13·35	474·37	13·25
392·46	13·33	435·50	13·37	475·46	13·23
393·49	13·33	437·50	13·49	478·48	12·96
397·46	13·60	438·37	13·53	479·42	12·80
400·40	13·54	439·31	13·57±	480·48	12·54
401·44	13·70	441·37	13·54	481·34	12·91±
402·40	13·37	445·36	13·23±	482·35	12·84
403·44	13·42	446·31	13·23±	483·34	12·91
404·46	13·36	447·49	13·31	485·34	13·0 ±
405·43	13·64	448·34	13·31	487·25	13·05
406·45	13·64	449·37	13·77	488·29	13·02
407·39	13·42	451·43	13·57	490·43	13·06
408·42	13·35	455·42	13·37±	491·24	13·08
415·47	13·57 ±	458·37	13·37	495·35	13·30
416·36	13·33	459·27	13·29	496·36	13·35
417·40	13·34	460·29	13·25	497·35	13·42
418·44	13·52	464·46	13·56	498·28	13·36

J.D. [2424]	Mag.	J.D. [2424]	Mag.	J.D. [2424]	Mag.
499·35	13·40	523·44	13·13	572·40	<13·0
501·34	13·13	525·41	13·23	578·36	13·67
502·29	12·96	526·25	13·05	584·37	13·67
506·30	12·74±	527·39	13·28	585·38	<13·5
508·27	12·59	528·25	13·31	596·38	<13·67
1926. Observations begun at Herne.		551·36	12·76	600·33	13·57±
		552·37	13·00	602·37	13·52±
519·25	13·08	555·40	13·3 ±	605·33	13·62±
520·26	13·08	560·37	13·28	607·36	13·62
522·25	13·08	563·44	13·64		

The magnitudes of the comparison stars are those given by Seares as used by Barnard and Steavenson.

All observations from J.D. 519 to 585 were made with the 5-inch refractor and should be regarded as second class, as the light grasp of the instrument is hardly adequate to deal satisfactorily with so faint an object.

Civil Date.	Julian Date.	Civil Date.	Julian Date.
1925 July 1	2424 333	1925 Dec. 1	2424 486
Aug. 1	364	1926 Jan. 1	517
Sept. 1	395	Feb. 1	548
Oct. 1	425	Mar. 1	576
Nov. 1	456	Apr. 1	607

Gorse Cliff School,
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A New Type of Measuring Engine for Photographic Plates.

By Frank Schlesinger.

The work of reobserving the zones of the *Astronomische Gesellschaft Catalogue*, now in progress at Yale Observatory, involves the identification on the plates of thousands of stars in each zone. To do this quickly and with certainty we have tried a number of devices. The engine here described was developed and constructed to facilitate these identifications, and it leaves little or nothing to be desired for such an application, enabling us to point quickly at an image by means of its right ascension and declination given in the Gesellschaft zone; or conversely to read directly from the plate the right ascension and declination of any object thereon.

We constructed this engine in our shop in rather rough form as this was sufficient for our immediate needs; but the principle involved applies to a more accurate engine, and I shall describe it here as though it were thus constructed.

A photographic plate is a central projection of a portion of the

celestial sphere ; that is, points on the plate are obtained by prolonging the radius of the sphere through the corresponding points on the latter's surface, the plate being tangent at a point called its base. Great circles on the sphere are projected into straight lines on the plate. Thus all stars having the same right ascension lie on straight lines which radiate from the projection of the pole upon the plate. In the diagram, B is the base of the plate and P is the projection of the pole, the distance BP being equal to the focal length of the telescope divided by the tangent of the declination (δ) of the base. If S is a star image, X and Y are the usual rectangular co-ordinates referred to B as origin and BP as the axis of Y. The angle A is given by

$$\tan A = X/(\cot \delta - Y).$$

We have also the well-known relation

$$\tan \Delta\alpha = X/(\cos \delta - Y \sin \delta),$$

where $\Delta\alpha$ is the difference in right ascension between S and B ; and consequently we have rigorously

$$\tan \Delta\alpha = \tan A \operatorname{cosec} \delta \quad . \quad . \quad . \quad (1)$$

or developing to include terms of the sixth order

$$\Delta\alpha = A \operatorname{cosec} \delta - \tfrac{1}{3}A^3 \operatorname{cosec}^3 \delta \cos^2 \delta + \tfrac{1}{15}A^5 \operatorname{cosec}^5 \delta \cos^2 \delta (1 + 2\cos^2 \delta) \quad (2)$$

We see that A is proportionate to $\Delta\alpha$ except for terms of the third and higher orders.

A parallel of declination on the celestial sphere, that is the locus of points having the same declination, is projected on the plate as a conic ; an ellipse if the sum of the declinations of the base and the star ($\delta + \delta_1$) exceeds 90°, an hyperbola if this sum is less than 90°. The equation of these declination conics is

$$\sin^2 \delta_1 (1 + X^2 + Y^2) = (\sin \delta + Y \cos \delta)^2 \quad . \quad . \quad (3)$$

where δ_1 is the variable parameter. Differentiating this twice with respect to X and remembering that $\frac{dY}{dX}$ is zero when X is zero, we obtain for the radius of curvature at the vertex

$$\rho = \frac{\sin \delta \cos \delta + Y \cos \delta}{\sin^2 \delta_1} - Y \quad . \quad . \quad . \quad (4)$$

Or substituting from (3)

$$\rho = \frac{\cos \delta}{\sin \delta + Y \cos \delta}(1 + Y^2) - Y \quad . \quad . \quad . \quad (5)$$

Consequently the declination conic that passes through the base (Y=0) has its centre of curvature exactly at the projection of the pole upon the plane of the plate.

If, therefore, the positions of stars are measured in polar co-ordinates with P as a centre we obtain close approximations to their right ascensions and declinations.

In the diagram, M is a reading microscope sliding along the rigid arm MP, its position being read on a scale or a long micrometer screw. This arm is pivoted at P, and may be rotated around that point by means of a worm-sector WW and a micrometer worm TH. To locate a star image on the plate the worm is turned and the microscope moved along the arm until the image is bisected by both micrometer wires in M; the readings at M and H are then recorded. The scale on which the position of M is read should be long enough to cover the whole extent of declinations that it is desired to observe, or else this scale and the microscope should be constructed in a unit that can be attached to the rotating arm at any distance from P.

If we define D as the difference between PS and PB, we have

$$\cot\delta - Y = (\cot\delta - D)\cos A \quad . \quad . \quad . \quad (6)$$

or, all fourth order terms included,

$$Y = D + \frac{A^2}{2\tan\delta} - \tfrac{1}{2}DA^2 - \frac{A^4}{24\tan\delta} \quad . \quad . \quad . \quad (7)$$

Combining this with (2) and the well-known development

$$\begin{aligned} Y = {} & \Delta\delta - \tfrac{1}{2}\Delta\alpha^2 \sin\delta\cos\delta + \tfrac{1}{2}\Delta\alpha^2\Delta\delta(\cos^2\delta - \sin^2\delta) \\ & + \tfrac{1}{3}\Delta\delta^3 + \tfrac{1}{12}\Delta\alpha^4(6\cos^2\delta - 1)\sin\delta\cos\delta \\ & - \tfrac{1}{2}\Delta\alpha^2\Delta\delta^2 \sin\delta\cos\delta \quad . \quad . \quad . \quad . \quad . \quad (8) \end{aligned}$$

we obtain

$$\Delta\delta = D - \tfrac{1}{2}DA^2\cot^2\delta - \tfrac{1}{3}D^3 + \tfrac{1}{2}A^2D^2\cot\delta + \tfrac{1}{8}A^4\cot^3\delta \quad . \quad (9)$$

or, all fourth order terms included,

$$\Delta\delta = D - \tfrac{1}{2}X^2Y - \tfrac{1}{3}Y^3 + \tfrac{3}{8}X^4\tan\delta \quad . \quad . \quad (10)$$

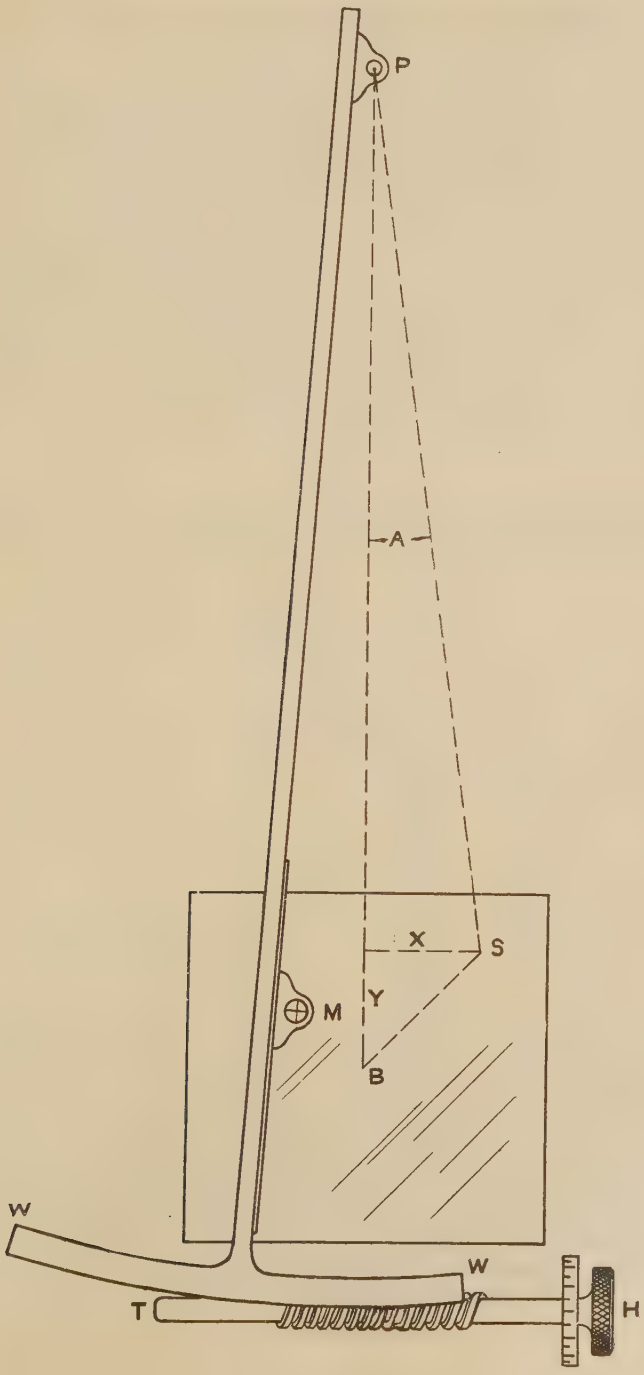
We learn from (9) and (10) that D differs from $\Delta\delta$ only by terms of the third and higher orders.

From (10) we see also that the third order corrections do not contain δ and depend only upon the rectilinear co-ordinates of the stellar image. For plates of the size employed for the Astrographic catalogue the maximum value of X or of Y is 1°, and consequently the third order terms cannot exceed 0″·91. The fourth order term is equal to 0″·0072 $\tan\delta$, for X = 1°, and may be neglected on plates of this size.

As the third order terms can never exceed 0″·91, they may conveniently be applied by making a diagram on the same scale as the plate itself, showing by contour lines the amount of the correction in various parts of the plate. If this is placed parallel to the plate, a long and light pointer attached to M and reaching to the diagram will automatically indicate the correction, and this can be recorded when the measurement itself is made. It is possible to use the same correction diagram for all the plates taken with any particular camera, but the description of the details of such an arrangement would be out of place here.

Equation (2) may be put in the form, third order terms included,

$$\Delta\alpha = A\,\mathrm{cosec}\,\delta - \tfrac{1}{3}X^3\sec\delta \quad . \quad . \quad . \quad (11)$$


P
A
X
S
Y
M
B
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W
T
H

The error in right ascension (measured in arc of a great circle) caused by the neglect of third order terms is therefore

$$-\tfrac{1}{3}X^3,$$

which is again independent of the declination and might be read from a correction-diagram like that just described, or from the same diagram with contour lines of a different colour from those used for the declination. For plates of the astrographic size, however, this correction may be omitted. To be sure its maximum amount is $0''\cdot36$, and this is usually more than we should wish to neglect; but if we omit it altogether the least-squares solution (or other process), by which the scale of the plate is determined from the comparison stars, will automatically take into account the greater part of this term. Let us assume that the comparison stars are uniformly distributed on the plate, then the least-square solution will so determine a (the scale-value correction) as to make a minimum the quantity

$$\int_0^{+\cdot01745}(aX-\tfrac{1}{3}X^3)^2dX,$$

the limit being 1°, expressed in radians. Performing the integration, differentiating with respect to a and equating the result to zero, we obtain

$$a=+\cdot0000609,$$

and consequently the neglect of the third order term involves the errors

$$+\cdot0000609X-\tfrac{1}{3}X^3.$$

The amounts of these errors for various values of X are as follows:

X.	Error.	X.	Error.
°	″	°	″
0·0	·00	0·6	+·05
0·1	+·02	0·7	+·03
0·2	+·04	0·8	−·01
0·3	+·06	0·9	−·07
0·4	+·06	1·0	−·14
0·5	+·06		

We see then that on plates of this size we may assume without serious error that A is proportionate to $\Delta\alpha$. This conclusion would not be modified if some other process than least-squares were employed for deducing the scale value from the comparison stars.

The necessity for multiplying A by cosec δ in order to obtain $\Delta\alpha$, involves, in general, no additional labour; for with ordinary engines we must multiply readings on scales and screws by appropriate factors that reduce them to arc or time, and in the present case the two multiplications can be combined into a single operation.

We next inquire what errors are incurred if the centre of rotation P is not at the correct distance from the base of the plate. We do not need

to consider in this connection the error due to the uncertainty in determining the position of the base, since this gives rise to errors that are inherent in the plate and do not depend on the method of measurement. We need only study the effect of an uncertainty in the focal length of the telescope at the time the plate was taken. If the error in the focal length is ΔF then the point P is misplaced by

$$\Delta F \cot \delta,$$

and the error in A is

$$\Delta A = A\Delta F + AD \tan \delta \Delta F.$$

The first term is large, but may be disregarded since it is a pure scale-value error. The second term, expressed in seconds of arc and substituting X and Y for A and D, is

$$XY\Delta F \sin \delta \tan \delta.$$

This error is small except for plates in very high declinations ; thus if ΔF is one millimetre, then the amount of this error in the *corners* of astrographic plates is as follows :

$\delta = 40^\circ$	$0''.010$
50	0·017
60	0·027
70	0·047
80	0·10
85	0·21

The corresponding error in the other co-ordinate D is

$$\Delta D = \Delta F \cot \delta - \frac{\Delta F}{2} \cot \delta \, A^2.$$

The first term is a constant and therefore introduces no real error. The second term may be put into the form

$$-\tfrac{1}{2}\Delta F \tan \delta X^2.$$

It has the following values on the *edges* of plates of the astrographic size, if ΔF is again one millimetre :

$\delta = 40^\circ$	$0''.008$
50	0·011
60	0·016
70	0·025
80	0·052
85	0·104

We see that on these plates F may be uncertain to the extent of several millimetres unless the plate is one of high declination. It ought always to be possible to determine F within a millimetre. For example, if the distance between two stars whose images appear on opposite edges of the plate is known within $1''$, then F can be found within about half a millimetre.

To sum up : *the geometry of this engine is such that right ascensions and declinations can be read off with practically no more labour than is required to secure rectilinear co-ordinates with engines of the ordinary type.*

The actual construction of such an engine allows much choice in the matter of details. For example, the worm-screw may be replaced by an arc of a graduated circle and may be read with a reading microscope ; or, again, an eye-piece scale, such as has been employed at Greenwich, Oxford, and elsewhere to measure astrographic plates, can readily be adopted here.

The greatest disadvantage of this form of engine is its size ; thus if it were employed to measure plates at $\delta=45°$ the length of the arm PM would be equal to the focal length of the telescope. With astrographic instruments, whose length is 3·4 metres, it would hardly be practicable to use the engine at lower declinations than 45° : and even with the shorter cameras that are now in use at many observatories it would seldom be advisable to measure at declinations lower than 30°. At first sight the use of a long arm appears to introduce the difficulty of changes in temperature. But if the arm is constructed of the same metal and with approximately the same cross-section as that which unites the plate with the socket under P, changes of temperature are made harmless. For example, if iron is used a difference of 1° C. changes the length of a 3·4 metre arm by 0·4 mm. ; as the plate will move by the same amount the effect is equivalent to an error in ΔF, similar to the one that we have discussed. In any case the observer will assure himself, as is customary with all other forms of measuring engines except that in which the reseau is employed, that the plate and scale have maintained their relative position ; this can best be done by measuring several " zero " stars at the beginning and the end of each session of measurement.

Considering the geometrical aspect alone of this type of engine, it is clear that we do not need actually to connect M with P by means of a long arm, but may instead employ a system of links that will permit the point M to move in accordance with the conditions. Whether such systems can be constructed in a way that would be mechanically satisfactory is doubtful, and this question would probably have to be answered by constructing an experimental engine.

I have discussed this engine with special reference to plates of the astrographic size because these are, generally speaking, the largest that are in use for the determination of precise positions. In fact I believe the only important exception to this statement is presented in the work now going on at Yale Observatory where plates of much larger angular dimensions are being employed to reobserve the Gesellschaft stars. With plates considerably larger than the astrographic, this type of engine could not be used without sacrificing something of its accuracy or something of its convenience. However, for the location of objects that cannot or need not be measured with the highest precision (such as asteroid trails and images of comets) this engine could be used to advantage on plates much larger than two degrees square.

Yale University Observatory
1926 *March* 15.

Note on the Radiation from a Pulsating Star. By J. J. M. Reesinck.

(*Communicated by the Secretaries.*)

In M.N., 1926 January, Dr. Jeans considers the radiation from a pulsating star, and arrives at the conclusion that the luminosity and the temperature should be in phase with r. Professor Eddington has demonstrated (*M.N.*, **79**, 17, 1918) that a difference in phase can only occur in the outer layers. An investigation on the atmospheres of the Cepheids will soon be published by the author. Here I make some remarks relating to Dr. Jeans' article.

Dr. Jeans' treatment of the equations (14) and (15) becomes illegitimate in the outer layers. On page 88 it is proved that λ is small. But now Dr. Jeans excludes the possibility of T_2 becoming large of the order of $\frac{1}{\lambda}$. So, in connection with (10), any difference in phase is excluded *a priora*.

At the boundary $\frac{d}{dr}(r^2H)=0$, but r^2H remains positive. Neglecting in (14) and (15) the terms with λ, but not those with λT_2, the equations become :

$$-H\frac{d}{dr}\left[(6+n)\frac{T_1}{T}+4\frac{r_1}{r}-\frac{\rho_1}{\rho}+\frac{\frac{dT_1}{dr}}{\frac{dT}{dr}}\right]=\rho c_v q\lambda T_2$$

$$-H\frac{d}{dr}\left[(6+n)\frac{\lambda T_2}{T}+\frac{\frac{d(\lambda T_2)}{dr}}{\frac{dT}{dr}}\right]=-\rho c_v q T_1+qP\frac{\rho_1}{\rho}.$$

Let x be the optical depth. Suppose, for convenience, that the absorption-coefficient remains constant during the pulsation (although it may vary with x). The equations reduce to :

$$H\frac{d}{dx}\left[3\frac{T_1}{T}+\frac{\frac{dT_1}{dx}}{\frac{dT}{dx}}\right]=-\frac{c_v q}{k}\lambda T_2 \quad . \quad . \quad . \quad (14a)$$

$$H\frac{d}{dx}\left[3\frac{T_2}{T}+\frac{\frac{d(\lambda T_2)}{dx}}{\frac{dT}{dx}}\right]=\frac{c_v q}{k}T_1-\frac{q}{k}\frac{P}{\rho^2}\cdot\rho_1 \quad . \quad . \quad . \quad (15a)$$

In an atmosphere we have, according to Schwarzschild, $H=\mu T_b^4$,

T_b being the boundary-temperature. Further, $T=T_b\sqrt[4]{1+x}$, and therefore

$$\frac{dT}{dx}=\tfrac{1}{4}T_b(1+x)^{-\frac{3}{4}}.$$

$$\frac{dT_1}{dx}=T\frac{d}{dx}\left(\frac{T_1}{T}\right)+\frac{T_1}{T}\frac{dT}{dx}.$$

$$\frac{P}{\rho}=\frac{1+bx}{(b-1)x}hT_1.$$

h, the gas-constant, is of the order of c_v; b, the ratio of the force of gravity to the force of radiation-pressure, is a number larger than 1.

Substituting this in the differential equations (14*a*) and (15*a*), these become:

$$\frac{d^2}{dx^2}\left\{(1+x)\frac{T_1}{T}\right\}=-\frac{c_vq}{4\mu T_b{}^3k}\cdot\frac{\lambda T_2}{T}\sqrt[4]{1+x}$$

$$\frac{d^2}{dx^2}\left\{(1+x)\frac{T_2}{T}\right\}=+\frac{c_vq}{4\mu T_b{}^3k}\cdot\frac{T_1}{T}\sqrt[4]{1+x}-\frac{hq}{4\mu T_b{}^3k}\cdot\frac{1+bx}{(b-1)x}\cdot\frac{\rho_1}{\rho}\cdot\sqrt[4]{1+x}.$$

The factor $\frac{c_vq}{4\mu T_b{}^3k}$ is of the order of 10^{-5}; therefore, unless x is very large or very small, we may neglect the right-hand members of (14*a*) and (15*a*). It appears that the layers where the right-hand members become of some importance contribute hardly anything to the radiation or to the line-absorption. So the general solution for the boundary is, with sufficient accuracy:

$$\frac{T_1}{T}=A+\frac{B}{1+x}$$

$$\frac{\lambda T_2}{T}=C+\frac{D}{1+x}.$$

Without drawing any positive conclusion, I draw attention to the fact that Dr. Jeans' analysis is not sufficient to prove the absence of a difference in phase, between r and T, in the outer layers.

Unusual Bright Filaments on the Sun on 1926 February 22.
By T. Royds, D.Sc. (Plate 2.)

On 1926 February 22 the spectroheliograms obtained at Kodaikanal showed near an active sunspot the development of remarkable bright filaments which were much brighter than ordinary flocculi. I have not been able to find a similar instance in the whole of the Kodaikanal record of spectroheliograms extending from 1904, and I am not aware of any having been published by other solar observers. It would appear,

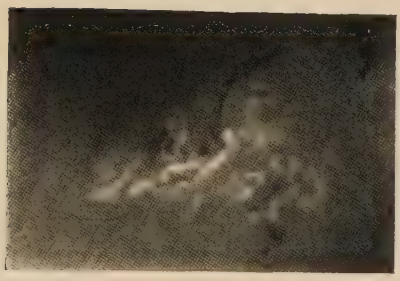


FIG 1. $8^h\ 3^m$

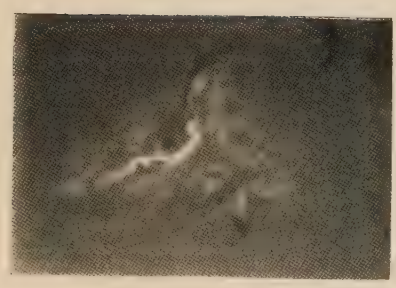


FIG 2. $8^h\ 8^m$

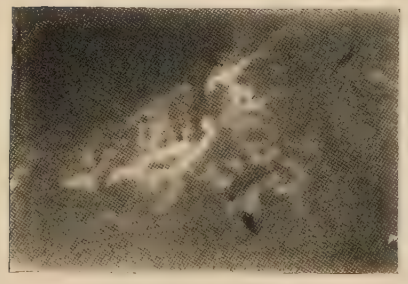


FIG 3. $8^h\ 36^m$

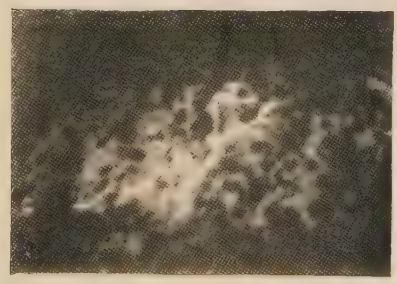


FIG 4. $9^h\ 0^m$

This line represents 9′ or 400,000 km.

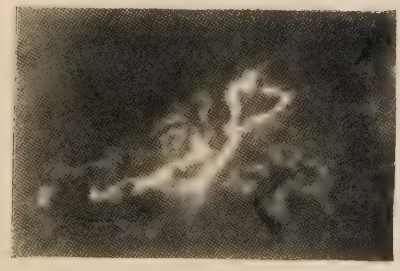


FIG 5. $9^h\ 4^m$

Figs. 1, 2, 3, and 5.—H_α Spectroheliograms.
Fig. 4.—K_{23} Spectroheliogram.
North is at the top.
East is on the right.

Kodaikanal Spectroheliograms on 1926 February 22,

therefore, that the phenomenon, which is illustrated in the accompanying plate, is a very unusual one.

Spectroheliograms were begun at $7^h\ 51^m$ I.S.T. (G.M.T.$+5^h\ 30^m$). Fig. 1 is an enlargement from the first H_α spectroheliogram at $8^h\ 3^m$ of a region near the active spot near the centre of the figure. The spot's position is 23° N. and 9° W. of the central meridian. Fig. 1 exhibits, in addition to dark markings and the usual bright flocculi, some parts of the flocculi which are much brighter than the rest, but this is not very exceptional. What is very unusual is to see these brighter parts develop into such intense and extensive bright filaments as exhibited in the later photographs.

By $8^h\ 8^m$, fig. 2, there is considerable growth in length and intensity; at $8^h\ 36^m$, fig. 3, there is further development, the more westerly branch now actually touching the sunspot, whilst the large dark marking seen to the north in figs. 1 and 2 has disappeared in fig. 3. Since the dark marking is restored, though in modified form and extent, in a spectroheliogram taken at $8^h\ 41^m$ with the slit set on the red edge of the H_α line, its apparent disappearance from fig. 3 should probably be attributed to Doppler displacement of the darkened H_α line. The dark marking reappears considerably changed in form in the spectroheliogram taken with the slit central on the H_α line at $10^h\ 24^m$ and is maintained thereafter.

Fig. 4 is a calcium K_{23} spectroheliogram taken at $9^h\ 0^m$. This spectroheliogram was intentionally under-exposed to prevent the ordinary flocculi from developing into the dense white masses familiar to all who have seen calcium spectroheliograms. In consequence of the under-exposure, the ordinary flocculi appear in only medium brightness, but the relative brilliance of the bright filaments is made clear.

The maximum development in brilliancy of these filaments was reached at $9^h\ 4^m$, shown in fig. 5. Subsequently the filaments persist with some changes of form, but become less brilliant. They were still striking in the last spectroheliogram taken at $15^h\ 6^m$.

The bright filaments described were due, of course, to brilliant reversals of the H_α and K lines. The reversals in the H_α line were examined visually in the spectroscope; the reversal was not, in general, displaced, but the dark H_α line was displaced locally by over 2 Å. Mr. S. Balasundaram Iyer, Assistant, making visual observations of disc phenomena with another telescope, found that near the umbra of the spot the D_1, D_2 and b lines were also reversed; D_3 was dark to the north of the spot.

There was very little movement of the bright filaments over the surface of the sun, the maximum velocity being only 1·4 km./sec.

On Some Relations between the Distribution of the Stars in Space and Mean Colour-Index or Spectrum. By G. Shajn.

(Communicated by the Secretaries.)

1. During the past twenty years it has been established that with increasing apparent magnitude the mean colour-index increases. Although the systematic errors are large in determinations of this kind, it appears to be very probable that such a relation between the colour and apparent magnitude really exists. On the other hand, the relative frequency of the stars of spectral types F, G, K, M increases with decreasing apparent brightness.* A similar relation between apparent magnitude and colour or spectrum should be expected for the stars belonging to clusters; here the relation between the colour and m is approximately the same as that between colour and absolute magnitude, and we know that the smaller the luminosity, the yellower is the colour.

Proceeding from our conceptions of a definite outer limit of the stellar system we may, as a first approximation, adopt that beginning from a certain apparent magnitude, for instance 15, there are no stars of high luminosity.† Beyond this limit the mean absolute magnitude and colour-index must therefore increase with decreasing apparent brightness. To explain the relation between colour and apparent brightness of the stars brighter than this limiting magnitude three hypotheses may be proposed—selective absorption of light in space, non-homogeneity in stars as regards spectrum or luminosity in different portions of space, and gradual thinning-out of the stellar system.

2. In this note I wish to show that proceeding from the density and luminosity functions found by Kapteyn and van Rhijn ‡ it is possible as a geometrical consequence of such a distribution of stars in space to deduce results bearing on the question under consideration.

From valuable and extensive data Kapteyn and van Rhijn derived the density and luminosity functions which determine the arrangement of the stars in space. Although the authors call their results provisional, they think it unlikely that more complete data will essentially modify them. They make two assumptions—(1) that there is no appreciable extinction of light in space, and (2) that the luminosity-curve does not change with the distance. It should be remembered that the density and luminosity functions are derived for all spectral types together. No doubt further investigations will lead to changes in the constants of both the luminosity and density functions, but their character will remain the same.

If the distribution of the stars in space were uniform, the star-ratio would be $2{\cdot}512^{\frac{3}{2}}$ or 3·98; but the density of stars with increasing distance decreases, and therefore the star-ratio is smaller than 3·98. If the arrangement of stars in space were uniform, and if the luminosity-curve were the same for all distances, the ratio of the number of white stars to that of yellow-red ones would be the same for each apparent

* Shapley and Cannon, *Circular of the Harvard College Observatory*, 226.
† Hertzsprung, *Ap. J.*, **42**, 337.
‡ Kapteyn and van Rhijn, *Ap. J.*, **52**, 23.

magnitude. We know that the absolute magnitudes of white (B, A) stars lie in most cases between the limits $-2{\cdot}5$ and $+2{\cdot}5$, while that of yellow-red (F, G, K, M) ones lie between the limits $-2{\cdot}5$ and $+11{\cdot}0$. For the sake of simplicity and illustration we shall make the assumption that the absolute magnitude of all the stars of type B, A is $+1{\cdot}3$, and that of type F, G, K, M is $+7{\cdot}0$. The distances of stars of apparent magnitude $7{\cdot}0$ and of absolute magnitudes $+1{\cdot}3$ and $+7{\cdot}0$ are 138 and 10 parsecs respectively.* For galactic latitudes 40°–90° the density is constant up to 25·1 parsecs, therefore the star-ratio $\frac{N_7}{N_6}$ for yellow-red stars would be 3·98 and that for white ones would be smaller than 3·98. It is known that the numbers of white and yellow-red stars of magnitude 6·0 are sensibly equal. The fainter the apparent brightness, the more the relative frequency of yellow-red stars should increase. In reality, the phenomenon is much more complicated. Though the mean luminosity of yellow-red stars is smaller than that of the white ones, the results depend on the proportion of yellow-red giants in space.

3. For the solution of this problem we have to find the star-ratio for white and yellow-red stars separately. It is evident that

$$\frac{N_{m_2}}{N_{m_1}}=\frac{\int_0^\infty \rho^2\Delta(\rho)f(m_2+5-5\log\rho)d\rho}{\int_0^\infty \rho^2\Delta(\rho)f(m_1+5-5\log\rho)d\rho} \quad . \quad . \quad (1)$$

where N_m is the number of stars within the limits $m\pm\frac{1}{2}$, ρ the distance, $\Delta(\rho)$ the density function, and $f(M)$ the luminosity function. The star-ratio is easily obtained by a numerical integration based on the data of the density and luminosity functions. I propose here a simple formula for the integrals. Let ρ_k be the distance of a star of absolute magnitude M' ($M'+\frac{1}{2}$ is absolutely the faintest star) corresponding to the apparent magnitude m; for the distances ρ_{k+1}, ρ_{k+2}, . . . of the stars of absolute magnitude $M'-1$, $M'-2$, . . . which correspond to the same apparent magnitude m, we shall have

$$\rho_{k+1}=y\rho_k\,;\quad \rho_{k+2}=y^2\rho_k\,;\quad . \quad . \quad . \quad \text{where} \quad y=\sqrt{2{\cdot}512}.$$

It is not difficult to deduce for the value of star-ratio the following formula

$$\frac{N_{m+1}}{N_m}=y^3\frac{\Delta(\rho_{k+1})f(M')+y^3\Delta(\rho_{k+2})f(M'-1)+y^6\Delta(\rho_{k+3})f(M'-2)}{\Delta(\rho_k)f(M')+y^3\Delta(\rho_{k+1})f(M'-1)+y^6\Delta(\rho_{k+2})f(M'-2)}$$
$$\frac{+\ .\ .\ .\ y^{3(n-1)}\Delta(\rho_{k+n})f(M'')}{+\ .\ .\ .\ y^{3(n-1)}\Delta(\rho_{k+n-1})f(M'')} \quad (2)$$

where $M''-\frac{1}{2}$ is absolutely the brightest star.

It might be better to take the intervals of magnitude smaller, but the nature of the data does not seem to call for such refinement.

From this formula it is evident, if we assume $\Delta(\rho)$ constant, that $\frac{N_{m+1}}{N_m}$ does not depend on the luminosity-curve.

* Kapteyn and van Rhijn, *Ap. J.*, **52**, 23.

At present we do not know the luminosity-curve for the separate spectral classes. For this reason a numerical solution to the problem was deduced in the following way. The star-ratios were computed by a numerical integration for all spectral classes together, and for the white (B, A) stars separately. For the former the integration is based on the data of the luminosity and density functions of Kapteyn and van Rhijn.* For the apparent magnitude m and successive values of M′, M′−1, . . . we find ρ_k, ρ_{k+1}, . . . ; f (M) and $\Delta(\rho)$ are taken from *Ap. J.*, **52**, 30, 36.

4. The subsequent line of attack for the white (B, A) stars may be described briefly as follows. Adams and Joy found that the absolute magnitude of the stars of types B and A is a function of spectral subdivision and of the character of spectral lines (n and s). In this method it is assumed that there is no dispersion in absolute magnitude for the same spectral subdivision. Although we know that this assumption cannot be rigorously true, the method is satisfactory.

The values of absolute magnitude so obtained were used for the derivation of the luminosity-curve. For the derivation of the luminosity-curve the data of the Henry Draper Catalogue, which is complete to about $8^{m}{\cdot}25$, were employed. It is admitted that some slight uncertainty is introduced in the results on account of the fact that for the stars employed the character of the spectral lines was unknown, and an average between n and s was assumed. However, the results of Adams and Joy prove that the influence of the character of the lines is not very large for A stars and is almost negligible for stars from Bo to B5.

By means of the formula $M=m+5+5\log\pi$ for intervals of 0·4 magnitude the values of π have been computed for all the stars of the H.D.C. of spectral types O, Bo, B1, . . . A5 to $8^{m}{\cdot}25$. The detailed counts and results will be published elsewhere. The stars were arranged according to both absolute magnitude and parallax. It is seen that such a table furnishes a determination of the relative number of stars of different absolute magnitudes—that is, of the luminosity-curve. I have tried to represent the luminosity-curve so obtained by an analytical expression.

The luminosity-curve of white stars (Bo–A5) in its first approximation may be presented by a Gaussian error-curve

$$\frac{h}{\sqrt{\pi}}e^{-h^2(M-M_0)^2} \quad . \quad . \quad . \quad . \quad ; \quad (3)$$

where $h=0{\cdot}568$ and $M_0=+1{\cdot}56$.

It is doubtful if the Bo–A5 stars have an error-curve distribution, and it is admitted that some uncertainty is introduced in the results on this account, but it is of no serious consequence. The computed values of star-ratios for the Bo–A5 stars are based on the data of (3).

5. The values of star-ratios have been computed for the galactic latitudes 0° and 40°–90° separately from $m=7{\cdot}0$ to 12·0. The results are given in Table I.

* Kapteyn and van Rhijn, *Ap. J.*, **52**, 23.

TABLE I.

Star-Ratio.	$\beta=40°-90°$.		$\beta=0°$.	
	All the Spectral Types together.	Bo–A5.	All the Spectral Types together.	Bo–A5.
N_7/N_6	3·01	2·72	3·58	3·51
N_8/N_7	2·80	2·38	3·37	3·19
N_9/N_8	2·56	2·11	3·16	2·89
N_{10}/N_9	2·38	1·85	2·96	2·62
N_{11}/N_{10}	2·22	1·60	2·77	2·40
N_{12}/N_{11}	2·06	1·35	2·59	2·18

Strictly speaking, it is impossible to compare the computed values of star-ratios with the observed ones on account of the difference in the galactic latitudes. However, a rough comparison shows that the agreement between them is satisfactory.

In order to find the frequencies of white and yellow-red stars of successive apparent magnitudes 7·0, 8·0, . . . we require to know those for $m=6·0$, and we take the data given in *Harvard Circulars*, 240 and 248.

B, A.	F, G, K, M.	
0·51	0·49	latitude $\pm(0°–15°)$
0·34	0·66	„ $\pm(40–90)$

These data and the values of star-ratios (Table I.) allow us to find the frequencies of white (B, A) and yellow-red (F, G, K, M) stars of apparent magnitude 7·0, 8·0, . . . (Table II.).

TABLE II.

Magnitude.	$\beta=0°$.			$\beta=40°-90°$.		
	B, A.	F, G, K, M.	Mean Colour-Index.	B, A.	F, G, K, M.	Mean Colour-Index.
7·0	0·50	0·50	+0·48	0·30	0·70	+0·74
8·0	·47	·53	·52	·26	·74	·79
9·0	·43	·57	·56	·21	·79	·84
10·0	·38	·62	·62	·17	·83	·88
11·0	·33	·67	·68	·12	·88	·94
12·0	·28	·72	+0·74	·08	·92	+0·98

6. Tables I. and II. are self-explanatory. *With decreasing apparent brightness the number of white stars increases slower than that of yellow-red on s*, i.e. *the mean colour-index with increasing apparent magnitude increases.* This peculiarity is a consequence of the distribution of the stars in space. The phenomenon depends on the density function, and therefore on the galactic latitude (it is well known that the density function depends on the galactic latitude).

The comparison of the regions 0° and 40°–90° shows also another peculiarity. *In the Milky Way the relative frequency of white stars is larger than that for high latitudes for stars of the same apparent magnitude,*

that is, the mean colour-index of the stars in Milky Way is smaller than that in high latitudes. The stars in the Milky Way are "bluer." It is evident that this well-known observed phenomenon can be deduced as a consequence of the density and luminosity functions. It must be noted that the possibility of deriving the first of these two results as a consequence of any thinning-out of the system with distance was first suggested by Hertzsprung.* The data of *Harvard Circulars* 240 and 248 allow us to find an approximate value of the mean colour-index of white (B, A) and yellow-red (F, G, K, M) stars of magnitude 6·0. The colour-indices of different spectral types are taken from *Mount Wilson Contribution*, No. 226, as follows :

B.	A.	F.	G.	K.	M.
−0·26	0·00	+0·38	+0·76	+1·25	+1·80

The mean colour-index of white and yellow-red stars (m=6·0) is −0·12 and +1·08 for the galactic region and −0·02 and +1·07 for the region 40°–90° respectively. Finally, these data and the frequencies of Table II. allow us to find the mean colour-index of the stars of magnitude 7·0, 8·0 . . . (the fourth and seventh columns of Table II.), referred to the visual scale. Assuming a linear increase in colour with m we find

$$\text{C.I.}=+0\cdot48+0\cdot05(m-7\cdot0) \quad \text{galactic region}$$
$$\text{C.I.}=+0\cdot74+0\cdot05(m-7\cdot0) \quad \text{latitude}\pm(40°\text{–}90°).$$

No doubt these results are affected by several uncertainties (mean colour-indices for m=6·0 are to some extent arbitrary, the influence of absolute magnitude on the colour . . .).

7. In order to see whether the computed frequencies of white and yellow-red stars and their colour-indices are close to those observed, we give below some data from various sources.

TABLE III.

Frequency of Spectral Types in Selected Fields.†

m (Visual).	Latitude ±(0°–15°).			Latitude ±(40°–90°).		
	B, A.	F, G, K, M.	Mean C.I.	B, A.	F, G, K, M.	Mean C.I.
Brighter than 6·0	0·51	0·49	+0·47	0·34	0·66	+0·70
6·0–7·0	0·51	0·49	·47	0·23	0·77	·82
7·0–8·25	0·44	0·56	·55	0·11	0·89	·95

Frequency of Spectral Types (all Latitudes together).‡

m (Visual).	B, A.	F, G, K, M.	Mean C.I.
Brighter than 6·25	0·41	0·59	+0·61
6·26–7·25	0·35	·65	·67
7·26–8·25	0·28	·72	·77
8·26–9·25	0·28	·72	·76

* Kaptcyn, *Ap. J.*, **40**, 2 ; *Mount Wilson Contributions*, No. 83.
† H. Shapley, *Harvard Circulars*, 240, 248.
‡ H. Shapley and A. Cannon, *Harvard Circular*, 226.

*Frequency of Colour-Indices in Milky Way.**

m (Photogr.).	10.	11.	12.	13.	14.	15.
C.I. from −0·6 to +0·2	0·47	0·52	0·44	0·44	0·29	0·19
C.I. from +0·4 to +2·0	0·53	0·48	0·56	0·56	0·71	0·81

Frequency of Spectral Types in Harvard Standard Regions.†

m (Photogr.).	N. (Latitude +51°·2).		S. (Latitude −50°·2).		P. (Latitude +21°·9).		M. (Latitude 8°·8).	
7·0	0·21	0·78	0·40	0·60	0·42	0·57	0·60	0·39
8·0	·11	·89	·15	·85	·32	·68	·65	·35
9·0	·02	·98	·15	·85	·29	·70	·60	·40
10·0	·00	·99	·05	·96	·11	·90	·47	·52
11·0	·00	1·00	·00	1·00	·11	·89	·33	·66

For the last two series m is referred to the photographic scale, and the colour-indices have not been computed.

The disagreement between the observational data is large. On account of the provisional character of the density and luminosity functions and some other uncertainties affecting the computed values there is a reason to be satisfied by the agreement between the observed and computed values, though it seems probable that the computed increase in mean colour with apparent magnitude is too large.

8. It is of interest to compute the total amount of starlight from any area in the Milky Way (for instance, ten square minutes) in terms of the light of a star of magnitude 6·0. From the data of *Harvard Circular* 240 we find that in this area there are 0·0029 white and yellow-red stars of magnitude 6·0. It is easy then to find the number of stars of successive magnitudes 7·0, 8·0, . . . The calculation is based on the data of Table II. The addition of 0·4 (6·00−m) to the logarithm of these numbers gives the logarithm of the amount of light contributed by the stars of corresponding magnitude. The amount of light from an area in the Milky Way of ten square minutes in terms of the light of a star of magnitude 6·0 is given in Table IV.

TABLE IV.

Limits of Magnitudes (Visual).	B, A.	F, G, K, M.	Ratio.	Δm (Visual).	Δm (Photogr.).
From 5·5 to 10·5	0·0238	0·0305	1·28	+0·27	−0·53
,, 11·5	·0300	·0425	1·42	·38	− ·42
,, 12·5	·0351	·0557	1·58	·50	− ·30
,, 13·5 (extrap.)	·0391	·0692	1·77	·62	− ·18
,, 14·5 ,,	·0421	·0823	1·96	·73	− ·07

The numbers of the second and third columns are referred to the visual scale. The fifth and sixth columns show the difference in the

* E. A. Kreiken, "On the Colour of the Faint Stars in Milky Way . . .," from Seares' paper in *Ap. J.*, **61**, 123.

† Miss Leavitt, *Harvard Circular*, 230.

magnitudes between the amount of light of the stars of types F, G, K, M and that of B, A. In the computations of the sixth column it was assumed the mean colour-index of stars F, G, K, M to be +0·80. The intensely interesting question arises: What will be the integrated spectrum of the Milky Way? Table IV. shows that the integrated spectrum depends on the limiting magnitude which can be reached by the very long-continued exposure. In the light of these data it appears that Fath's result that the integrated spectrum of the Milky Way is of solar type is not so surprising. It is well known that the spectrum of many close binaries (the primary of type F, G, K, M and the companion of type A) is of composite character with predominant lines of solar type. In most cases the difference in the magnitudes (visual) of the components lies between the limits +0·3 and +2·0. Thus, it is to be expected that the integrated spectrum of the Milky Way, if the exposure is continued long enough, would be of composite character with the predominant lines of solar type.

The recent observation of the integrated spectrum of the Milky Way by Slipher is as follows: "The spectrum shows stronger hydrogen lines than belong to the solar type, and would be termed *composite* with the solar lines predominant." *

It is to be remembered that the density function of Kapteyn and van Rhijn for the remote portions of space is very uncertain. Notwithstanding we have deduced from it consequences which are in close agreement with the observations of Fath and Slipher. This suggests the observation of the integrated spectrum of the Milky Way with various exposures. The same result can be obtained if we observe the colour-indices of a small area in the Milky Way. These studies may throw a light on the structure of the universe in the remote portions of space.

9. We know that the proportion of late type giants in space is small, and this knowledge is based on the proper motions, parallaxes, and spectra of comparatively near stars. Formula 2 shows that the value of the star-ratio depends in the main on the stars of high luminosity. If the proportion of yellow-red giants were several times increased, the increase of mean colour-index with decreasing apparent brightness would be very insignificant. However, among the faint stars this phenomenon is very large, and so we may conclude that the proportion of late type giants in the remote portions of space is small as well as in our neighbourhood.

10. It seems to be very probable that there is a sharp limit to the stellar system, and this assumption leads to the conclusion that among the faintest stars no absolutely bright stars occur. The distances of stars of absolute magnitude −2·6 and apparent magnitude 12 and 15 are 8,300 and 33,000 parsecs respectively, and this is near to the limits of the stellar system in high and low galactic latitudes. From this limiting magnitude, in addition to the general increase of colour-index (mentioned above), there is another cause which increases this effect, viz. a decrease of the mean absolute brightness of stars. Thus, it is to be expected that beyond this limiting magnitude no stars of high luminosity (white stars and yellow-red giants) occur, and therefore the minimum colour-index

* Slipher, *Pub. of the American Astr. Soc.*, vol. iv., 114.

increases. However, as the giants of late types are redder than the dwarfs of the same spectral type, the increase of mean and minimum colour-index will be partly compensated. It may be concluded, from all we know, that in the Milky Way the limiting magnitude, beyond which the mean absolute magnitude, and therefore the minimum colour-index, commences to increase, is 4 to 5 magnitudes fainter than that for high galactic latitudes. On the other hand, the observation of the minimum and mean colour-index in different galactic latitudes may lead to the determination of the limits of the stellar system.

As an explanation of the phenomena under consideration other hypotheses may be also proposed (absorption of light in space, non-homogeneity of spectral or luminosity characteristics in different portions of space). However, from what we know, the first of these hypotheses cannot be accepted, and there are no reasons for thinking that the second effect is sensible.

No doubt the results are affected by several uncertainties, and some objections can be raised against them; but it seems improbable that more complete data will seriously modify them. Though the distribution of luminosities in certain selected groups of stars show some deviation from that predicted by the density and luminosity functions, it appears to be very probable that the general character of these functions will remain the same.*

Summary.

The object of this note is to show that several phenomena (which are in general agreement with the observations) can be deduced from the density and luminosity functions; some others are suggested.

With decreasing apparent brightness the number of white (B, A) stars increases more slowly than that of yellow-red (F, G, K, M) ones, *i.e.* the mean colour-index increases. The relative frequency of white stars is larger in the Milky Way than in high galactic latitudes, the apparent magnitude being the same.

The integrated spectrum of the Milky Way should be of "composite" character with predominant lines of solar type.

The proportion of late type giants in the remote portions of space as well as in our neighbourhood is small.

For the faintest stars the minimum colour-index should be increased.

The predominance of late type spectra among the faint stars does not mean that the proportion of late type stars increases with distance.

* Seares, *Ap. J.*, **59**, 11; Hertzsprung, *Bull. of the Astr. Inst. of the Netherlands*, **1**, 22; Kienle, *A.N.*, **218**, 119; Luyten, *L.O.B.*, **11**, 24; Luyten, *Ap. J.*, **62**, 8; Öpik, *Pub. de l'Observ. Astr., Tartu*, **25**, 2.

Simeis Observatory:
1926 *March.*

The Structure of the Solar Cluster. By Professor R. A. Sampson, F.R.S. (Plates 3–5.)

The implications which flow from the fact that, after temperature, the total mass of a star governs its luminosity are of cardinal importance. Eddington deduced this result as a theoretical formula, and afterwards found it closely confirmed by all the known stellar masses, including several cases to which he had not expected to apply it. We may, therefore, divorce the result from theory, and treat Eddington's theoretical table as a mere interpolation formula smoothing and connecting known facts. It will be convenient to speak of it as a formula, in view of its smoothing effect and the way in which it provides all the other elements when two are known; but I recall here that its basis is observation. Its use is independent of whether the theory is right or wrong. For that matter, in 1895, I myself deduced from a crude and obsolete theory of radiation the statement that it is the total mass upon which the radiation of a star depends.* Taking a suggestion from this, one might have written down the now known masses and reached the same interpolation curve, since the way in which the temperature factor enters can hardly be in doubt.

The feature of the result which confers such cardinal importance upon it, from a mathematical point of view, is the reduction of the physical elements of the problem from three to two—mass and temperature alone, the radial co-ordinate eliminating itself. We know that the luminosity of the star is determined by a simple formula expressing it in terms of the area of the disc and the surface temperature. One would have expected when the luminosity was viewed from the alternative point of view as radiative emission, particle by particle from the star's mass, that the distribution of the mass along the radius, together with associated variations of internal temperature, would appear in very complicated forms. That they do not, makes one suspect that a theory, even simpler than has yet been offered, may suffice to cover all we know. Be that as it may, for any star for which are given (1) the visual magnitude, (2) parallax, and (3) spectral class, we are now in a position to calculate all the elements in ordinary use for physical description, namely, mass, radius, mean density, and surface temperature, along with the distribution of the stars in space.

The gain in clearness and force in employing the latter terms in place of the former is enormous. It draws the matter into the meshes of a vast net of familiar, active, physical ideas. It is admitted that the translation from one set of data to the other is open to improvement. The term "surface temperature" is vague; still it is a great improvement in clearness and physical connotation on "spectral class," and the numerical equations between the two are now not likely to be upset. In the same way, it is very probable that luminosity cannot be expressed completely and precisely in terms of total mass and surface temperature, probably other qualifying factors enter; but it cannot be admitted that there are any such as seriously to distort the admitted relation.

* *Mem. R.A.S.*, **51**, 123.

Now, as remarked above, a survey of the stars in distribution, temperature, mass, and radius, is complete physically. It is all that we can know of the stars as a system. Such a survey will give the structure of our system uniquely and completely in ordinary physical concepts in a way that statistical summaries such as the "luminosity law" and other such laws have failed to do. And the basis for the calculations appears to be well enough established, so that no radical change in the conclusions need be apprehended.

These considerations set in motion the investigation described below. The outcome of the survey is, briefly, the proof of the existence, long suspected, of a solar cluster, and the description in detail of its structure.

This structure may be summarised in the statement:—If we superpose to the familiar spectral classification of the stars by temperature, a cross classification by mass, the groups that result possess definite distributions in space, which are here assigned.

We have at the Royal Observatory, Edinburgh, two MS. collections of parallax, with other particulars, compiled on cards by Dr. Gorakh Prasad. The first consists of trigonometric determinations, and is complete to 1925 January. It contains over 1600 members, but some 400 of these required to be rejected because the parallax was negative or the spectral class not known. The second is of determinations by the spectroscopic method. It is more extensive, but not at the present date quite complete. I have used the former collection for making the present survey. This survey may be regarded as preliminary; the material has been taken as it stands without attempting a criticism of the magnitudes and spectral classification or parallax determinations. It was, however, compiled with scrupulous care by Dr. Gorakh Prasad, and nothing has arisen to suggest distrust.

The visual magnitudes and spectral classes are, with very few exceptions, those of the Harvard scales.

I calculate for each star, from the given spectral class, parallax, and visual magnitude, the following particulars:—(1) bolometric magnitude, (2) absolute bolometric magnitude, (3) surface temperature, (4) angular radius, (5) linear radius, (6) mass, and (7) mean density.

Incidentally I would remark on the importance of the idea of bolometric magnitude which Eddington introduced in 1917 (*M.N.*, **77**, 605). Both visual magnitude and photographic magnitude are mixed measures in which the peculiarities of the measuring apparatus are still present. Bolometric magnitude represents the physical residue when such peculiarities are removed. Absolute bolometric magnitude gives the true radiative emission of the stars, integrated for all wave-lengths. Take with it the effective temperature, which gives the character of the Planck radiation curve, and we have a physical description which is absolute and complete for the star's emission.

The formulæ employed are appended. These are adjusted so that the Sun's temperature, mass, and radius are taken as units. The Sun's mean density would therefore not be the unit, but equal to $3/4\pi$. But where no confusion can result, the mean densities are also given as multiples of the Sun's mean density.

The conversion from visual to bolometric magnitude was made by

the table given by W. Rabe, *Ast. Nach.*, **225**, 223. This is a little more extended than Eddington's table in *M.N.*, **77**, 605, and Rabe shows that the two are sensibly equivalent. They are independent in derivation.

Let M be the absolute bolometric magnitude.
m, m_b be the apparent visual and apparent bolometric magnitudes.
α be the angular radius of the star in seconds of arc.
a be the linear radius, that of the Sun being the unit.
M be the mass of the star, that of the Sun being the unit.
ρ be the mean density, that of the Sun being the unit.
T be the effective temperature of the star, in ratio to that of the Sun.

As remarked above, the measures of a, M, ρ, are not on the same dynamical system; in fact we have

$$M=a^3\rho\ ;$$

but it is convenient to refer each quantity to the Sun, and no confusion will result.

The value of the Sun's effective temperature was taken at 6200° C. In the calculations all these quantities excepting M enter through their logarithms; for brevity I have written $\lambda\alpha$, λM, etc., to stand for $\log_{10}\alpha$, $\log_{10} M$, etc.

To convert spectral class into effective temperature I have used the Table VI. which I gave in *M.N.*, **85**, 237, but I remark that if this is plotted with λT and spectral class as co-ordinates, it is sensibly a straight line with slight deviations only at the two extremities; in that it agrees with the experience which Fechner's Law illustrates in so many instances, namely, that increments which are sensibly equal (as in the spectral scale) represent constant proportional steps in a physical quantity and not constant additive ones. I have disregarded the slight deviations at the two ends, and have made the conversion as though the plot were strictly a straight line. Thus I have used throughout for each tenth part of a spectral class, the increment

$$\Delta\lambda T=\cdot 0164,$$

fixing the zero by

$$\lambda T_{Go}=0.$$

The classes Ma, b, c are taken as equivalent to Mo, 1, 2, and the class Mo as equivalent to K6.

Measuring the luminosity by α^2T^4, or, again, by $10^{-0\cdot4m_b}$ we have

$$0\cdot4m_b+2\log\alpha+4\log T=\text{const.}\ ;$$

or adopting for the Sun,

$$\alpha=961'',\quad m_b=-26\cdot72,$$

we have

$$\lambda\alpha+2\lambda T+0\cdot2m_b=-2\cdot36 \qquad \text{(i)}$$

Again

$$a=\alpha d,$$

that is

$$\lambda a = \lambda\alpha - \lambda\pi + \text{const.},$$

and for the Sun

$$\alpha = 961'', \quad \pi = 206265'',$$

so that

$$\lambda a = \lambda\alpha - \lambda\pi + 2{\cdot}33 \quad . \quad . \quad . \quad . \quad \text{(ii)}$$

Also

$$\lambda\rho = \lambda M - 3\lambda a \quad . \quad . \quad . \quad . \quad \text{(iii)}$$

Eddington's Table I. (*M.N.*, **84**, 310) has been employed for calculating λM when M and λT are given.

This table gives a calculated bolometric absolute magnitude for the Sun of 4·56, which is too bright. In other cases also the curve departs somewhat from observation. The true curve of relation may be considered still under discussion. In the meantime it was thought best, as indicated above, to employ the table without modification as an interpolation formula, which represented observation with ample security for the purposes of the present paper.

The whole number of stars dealt with is 1202, divided as follows :

B	24	including	Bo to B_5
A	152	,,	B8 ,, A_5
F	217	,,	A8 ,, F_5
G	410	,,	F8 ,, G6
K	334	,,	G8 ,, K_5
M	65	,,	M
	1202		

Applying the formulæ above, the physical elements of each star were calculated, and the stars of each class were then distributed in groups, according to mean density. It appeared that steps of 0·5 in $\lambda\rho$ or $\times 3{\cdot}16$ in density, made a convenient division. The Group G (0·5) thus includes all stars classed in spectrum between F8 and G6, of which the calculated mean densities fall between antilog. *0·74* and antilog. *0·25*, or between 0·18 and 0·56 that of the Sun.* Each group, then, has a certain range in temperature, mass, radius, mean density, and luminosity, but not to any wide degree. The mean values for the groups are taken as characteristic of the group. Thus, the groups are treated as homogeneous physically, and are apparently sufficiently homogeneous for any conclusions that are here drawn for them. The latter are connected almost exclusively with the numbers of members found in the groups and the distribution of these members in space. Any features in the distribution which may declare themselves are a new contribution to the problem, and are in no way imported into it by the formulæ of calculation.

The schedules of the various groups are given at the end of this paper (see Schedule I.).

It is not necessary or practicable to give the details of each group.

* In tabular work negative numbers are denoted by italic figures.

One example will suffice. I give at the end of this paper (Schedule II.) the group G (0·0), which is the most numerous single group, containing 101 stars of densities between 0·56 and 1·74 times that of the Sun.

Examining this group it will be seen that its members are found at all distances. Counting the numbers found within and up to any assigned distance, we obtain a plot with well-defined features, which are repeated in kind in all the groups. The curve is of parabolic character near the origin, passes through an inflexion at a certain distance, and finally converges to an asymptotic limit. Evidently the density of occurrence falls off as we recede from the origin at a rate which outweighs the spread of volume. The numbers are so smooth as to invite expression by a mathematical formula; but if we use one, it is under the disadvantage of idealising a discontinuous phenomenon by a continuous representation. Choosing the first and simplest formula that presents itself, let there be a continuous distribution of density Ae^{-kD} at a distance D from the origin. The quantity of matter found within a sphere of radius D is

$$N=\int_0^D 4\pi AD^2e^{-kD}dD \qquad [\pi=3\cdot14\ \ldots\ ]$$

$$=N_u[1-e^{-kD}(1+kD+\tfrac{1}{2}kD^2)] \quad . \quad . \quad . \quad (1)$$

where N_u is the limiting value of N for $D\to\infty$.

Here

$$N_u=4\pi Ak^{-3} \quad . \quad . \quad . \quad . \quad (2)$$

Further, there is an inflexion in N, where $d^2N/dD^2=0$, *i.e.* where

$$0=4\pi Ae^{-kD}(2D-kD^2),$$

or for

$$D=\infty,\ 0,\ \text{or}\ \ 2k^{-1}\equiv D_i,\ \text{say} \quad . \quad . \quad . \quad (3)$$

The mean distance

$$=\int_0^\infty D\,.\,4\pi AD^2e^{-kD}dD/N_u=3k^{-1}=\tfrac{3}{2}D_i \quad . \quad . \quad (4)$$

The number found within the sphere of radius D_i

$$N_i=N_u(1-5e^{-2})=\cdot323\ N_u.$$

The number within

$$2D_i \quad =\cdot762\ N_u,$$
$$4D_i \quad =\cdot986\ N_u.$$

The constant A marks the central density; to give it a concrete meaning, suppose $k=0$, and imagine a cubic lattice of interval D_0 between nearest members, centred at the origin and extending indefinitely. The cube of side $2nD_0$ includes $(2n+1)^3$ members.

Dividing by the volume $(2nD_0)^3$ we have $A\to D_0^{-3}$ for $n\to\infty$, so that a given value for the constant A implies that if the density were uniform throughout, at its central value, the distance between adjacent members would be $D_0=A^{-\frac{1}{3}}$. Hence

$$D_0=D_i(\pi/N_u)^{\frac{1}{3}} \qquad [\pi=3\cdot14] \quad . \quad . \quad . \quad (5)$$

Now, suppose the actual number of members of a discontinuous group found within and up to a distance D to be represented by the formula for N given above. Adjust the two parameters N_u, k, so as to secure the best fit, or what is equivalent, D_0, D_i, and we have a representation of the distribution. Actually, D_i, N_u are found very readily by trial, the former as marking the distance at which the inflexion occurs, and the latter from the ultimate number of members. A table of the function $Z \equiv 1 - e^{-z}(1+z+\frac{1}{2}z^2)$ (*cf.* fig. 1) with argument

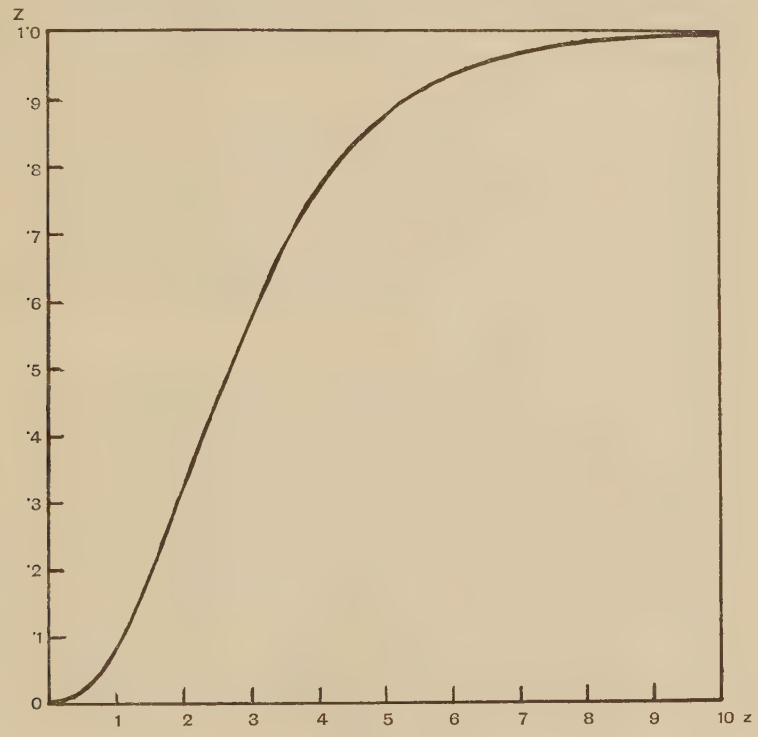


Fig. 1.

$z = kD = 2D/D_i$, permits immediate calculation of the numbers N. D_i, N_u, and D_0 were thus found for each of the groups which offered enough members to show the character of the distribution. Plate 3 shows the observed distribution for the groups. The continuous curves drawn among the plotted values of observed N show the comparison of observed and calculated numbers. There are two or three cases where the agreement is inferior. Such cases might be the expression of an eccentricity, the distribution being a little out of centre with the Sun. Otherwise it is good and sufficient throughout. Simple as the theory is, one could hardly expect to improve upon it.

The evident connected system of all these plots must be considered a very remarkable fact. Indeed, it can only be read as the result of a

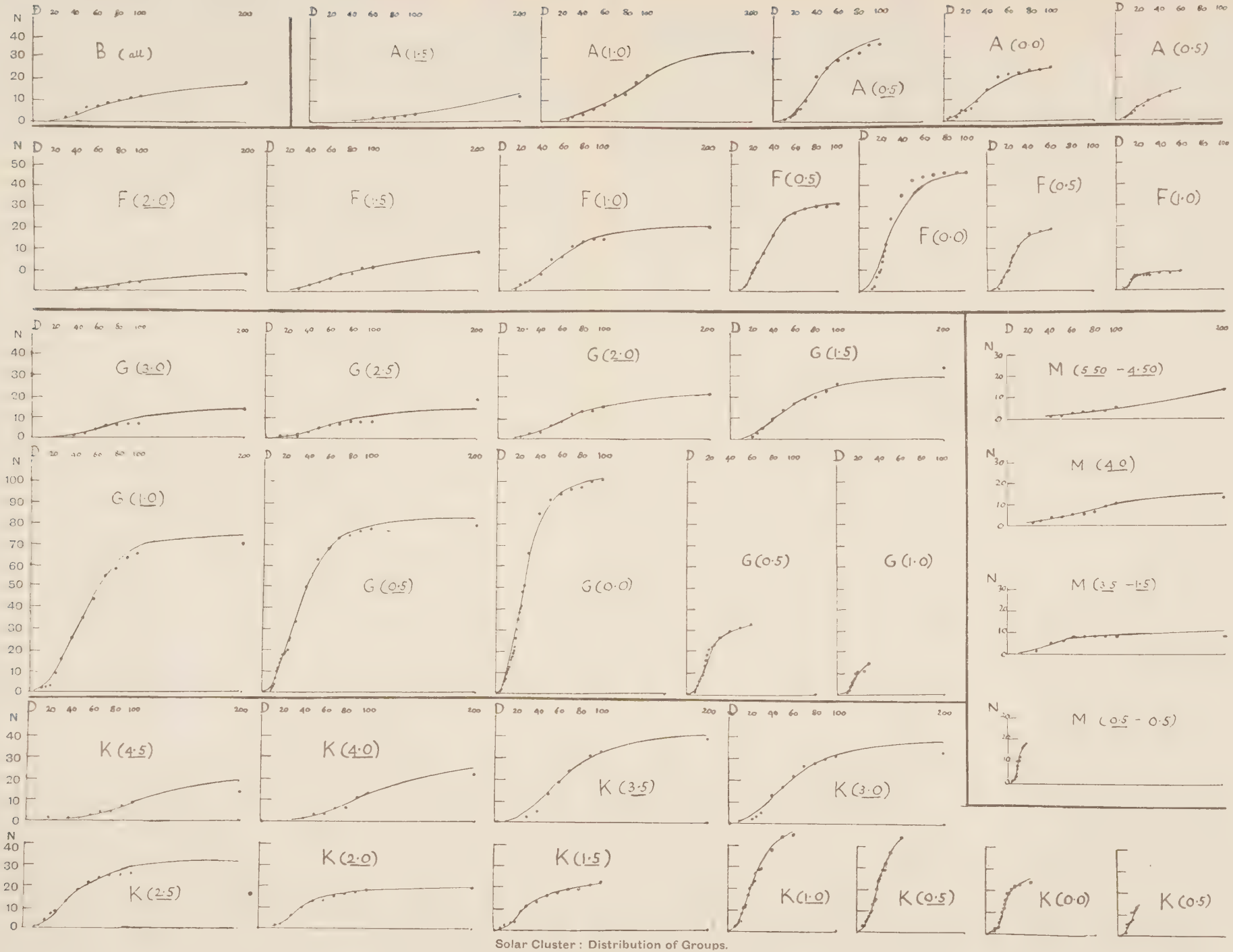
rule. It is so smooth that one seeks for some avenue by which it may have been imported tacitly into the presentation; but no such thing is found. The groups were collected in complete independence of each other. Certainly, each has been collected as being approximately homogeneous in mean density, which could not be done without the use of the mass formula; but, apart from the fact that the mass formula is sufficiently established, they might just as well have been collected as groups of the same luminosity, and this would have used only a formula about which no debate is possible. Hence the regularity and the connection of the groups are real effects and not artificial. They are the demonstration of the existence of a Solar Cluster, and therewith the description of its structure.

This matter is so important that I will put it from a different point of view with the endeavour to expose any cause, whether substantial or not, by which the obvious systematic character might be produced or enhanced. First, undue weight must not be given to the smoothness of the plots. N represents the number of cases found, up to and including a sphere of radius D. Hence, as D increases, N can never diminish, though it may fail to increase. This will contribute a certain smoothness which carries no meaning. But this must not be confused with the smooth progression of the numbers of successive groups which there is nothing to explain.

Another appearance that requires scrutiny is the uniformity of the whole system, by which all the graphs were calculable from a single formula. All the curves are "similar" to one another, but two parameters are disposable, and this allows some range in fitting. One may ask whether the collection of N really discriminates between a pure system and a mixed one. Light is thrown upon this by the treatment of the B-stars in the analysis above. As these are not numerous enough to define separate groups, they have all been run into one group, with the consequence that a wide range of mass and density is covered. Hence, D_0, D_i lose their specific meaning; but because all the B-stars are remote and their distribution open, a curve with definite values for D_0, D_i fits the combined group not too badly to carry a certain value. The result would be very different, however, if we ran all the F- or G-stars together. The distinctions which are now so sharp would be altogether obliterated and the structure lost.

Where reference is made to individual stars or small groups the question has little meaning, because it is not supposed that any machine-made stamp exists by which stars of one system can be sorted out from similar neighbours. Hence, as applied to a single homogeneous group, collected as in the foregoing pages, we may dismiss the enquiry; but it becomes significant again when we compare the groups with one another. For example, we may ask: Is the double maximum of the K-class, which characterises the Giant and Dwarf phenomenon, a feature that can be accounted for under one head, or an expression of the superposition of a second cluster upon the first? It could certainly bear the latter construction, and if so, its evolutionary meaning would disappear.

The essential feature of the cluster is the gathering round a centre.



Solar Cluster : Distribution of Groups.

For a distribution which had no relation to any centre we should have simply the number of stars increasing proportionately with the volume enclosed, or $N \propto D^3$. The plot of this relation has no inflexion and no finite limit. In practice a finite limit would be imposed on the observed members of such a distribution because, with constant absolute magnitude and increasing distance, they would fall in brightness until they passed out of the practical observing list. A finite limit means a horizontal asymptote, and a horizontal asymptote could necessitate an inflexion. Hence, practical limitations would transform the cubical parabola $N \propto D^3$ into a curve of the same type as shown in fig. 1. The question is: Does limitation by distance actually operate in the collection of cases dealt with here so as to produce the features remarked upon? We may answer this question as follows: The inflexions were found from the actual graphs for the groups. Where the inflexion occurs, the limitation of numbers and consequent clustering, whether a real or an artificial effect, has already made itself felt. Now, the mean absolute bolometric magnitude for each group is known, and we can calculate the apparent visual magnitude of the members which occur at the distance D_i belonging to it. We find the following table:—

TABLE I.

Mean Apparent Visual Magnitudes at Distance D_i.

Group.	B.	A.	F.	G.	K.	M.
5·0	..	..	..	..	..	3·5
4·5	..	..	..	..	2·9	
4·0	..	..	..	..	3·9	4·7
3·5	..	..	..	..	3·9	
3·0	..	..	..	3·4	4·4	
2·5	..	..	..	4·0	5·2	
2·0	..	..	4·0	4·9	6·0	
1·5	..	4·1	4·8	5·4	6·3	
1·0	3·0	4·4	4·7	6·4	7·0	
0·5	..	3·5	4·7	6·2	7·4	
0·0	..	4·2	5·3	6·7	8·5	9·3
0·5	..	4·5	6·3	7·1	9·0	
1·0	..	..	6·2	7·8		

Now it is certain that a false clustering which in reality was the effect of distance would operate at the same apparent magnitude upon all classes and groups. It is impossible to explain by such means the progressions shown in the table above. The table may be regarded as the demonstration that we are not dealing with mere visual extinction. It is very possible that the collections for the higher densities are modified by the distance effect, and fail to be fully representative, especially for K- and M-stars. Some of the plots have the appearance of being cut off before completion. As to how far such modification could be traced throughout, I held an open mind during the discussion. The trials required for fitting the theoretical curve to the observed plots

gave occasion for the enquiry. I was disposed to set the estimated N_u at a rather higher figure than the actual totals of stars recorded. As an outcome I found that a little allowance was certainly beneficial, but on the whole there was little evidence that it was necessary, and none at all that the curves were sensibly modified in consequence. Now N_u may be taken as referring to distances of $4D_i$ and upwards. My conclusion is that the plots are unexpectedly clear of any distance effect, and show almost purely the effect of true clustering.

Finally, I would refer to the possibility that those who drew up the observing lists exercised some selection which worked to the exclusion of small parallaxes. Now, whether such a selection was in conscious or unconscious use, I consider that it is proved to be ineffective in operation by the very large proportion of negative parallaxes that occur.*

It will naturally be asked what relations these groups exhibit to the galaxy. I have made the study for a few of the most numerous groups, and show the results below.

TABLE II.

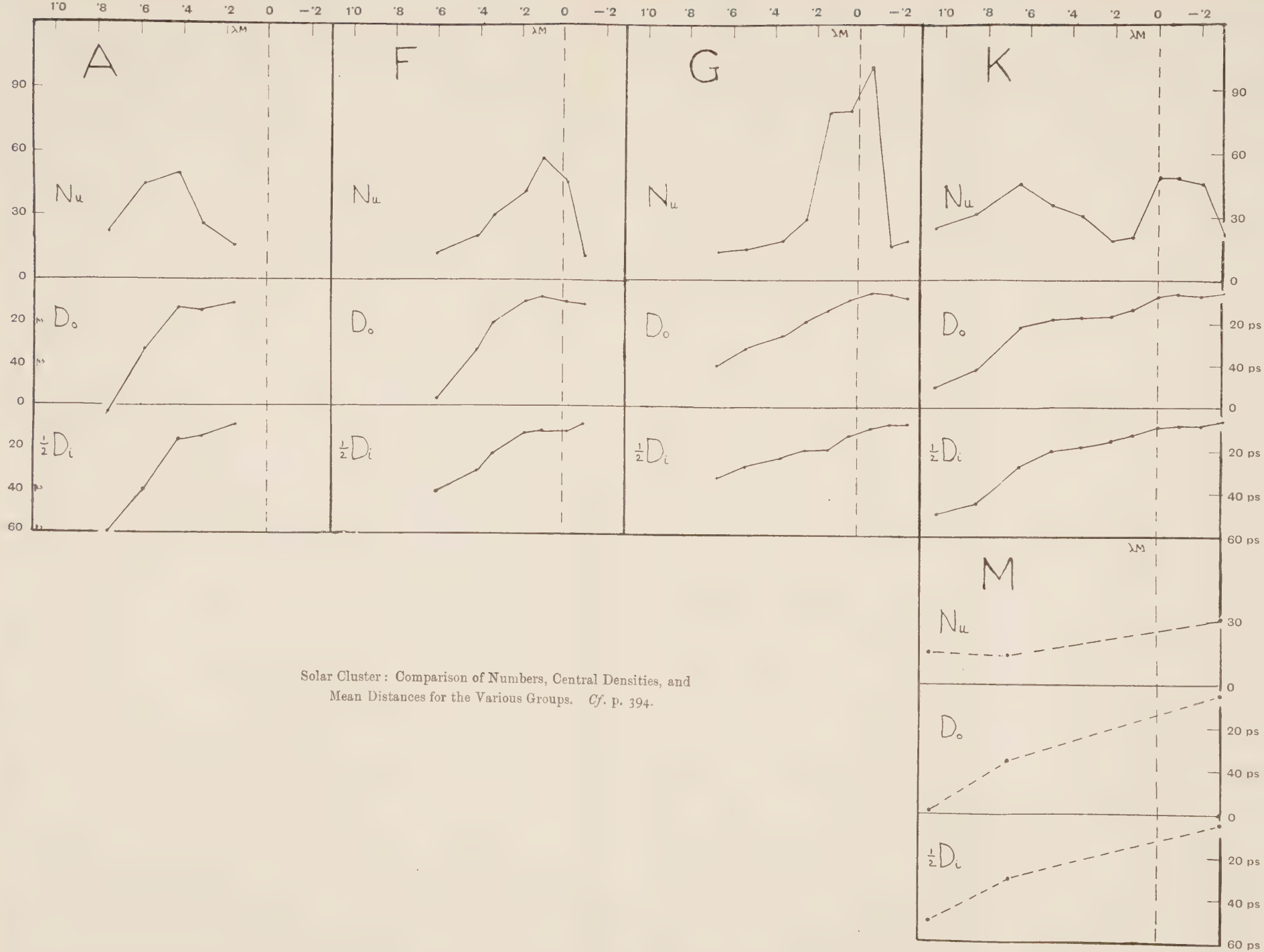
Galactic Latitude.	G (0·0) Number.	Density.	A (1·0) Number.	A (0·5) Number.	A (0·0) Number.	Sum.	Density.
90°.. 60°	3	11	1	0	1	2	8
60.. 30	16	22	2	6	1	9	13
30.. 0	27	28	12	7	13	32	33
0..−30	29	30	11	9	4	24	25
−30..−60	23	33	8	14	5	27	39
−60..−90	3	11	1	5	3	9	35

The result is negative. Very little, if any, condensation of the members towards the galaxy is shown. Equally, no spheroidal spread along the plane of the galaxy is shown.

I take now the consideration of the quantities D_0, D_i, and N_u, which specifically describe the groups. The values are exhibited in relation to one another in Schedule I., and graphically in Plate 4.

The mere facts of the Giant and Dwarf phenomenon declare themselves, as of course they must. The series is cut in two for the M-stars. There corresponds a double maximum in the K-stars. There is one great maximum in the G-stars, but whether it represents the two maxima of the K-stars merged together does not appear. To elucidate the origin of these features we may examine the progression of values of D_0, D_i, being the central span and the inflexional distance of each group. The latter is characteristic of the type of distribution; the former, in conjunction therewith, gives the number of stars that occur in it. It will be seen that neither quantity alone shows the double maximum. The ratio of the two reproduces very faithfully the actual numbers of occurrence for all classes. But, referring to formula (5), p. 394, this is no more than to say that the double maximum is shown by N_u. There is in this no clue to any meaning.

* For further considerations, see the *Addendum*, p. 410, containing replies to objections raised by the referees.



Solar Cluster: Comparison of Numbers, Central Densities, and Mean Distances for the Various Groups. *Cf.* p. 394.

Examining by eye the run of the charts of D_0, D_i, one is tempted to signalise as a characteristic feature a sudden change in gradient, whereby any of them could be well represented by two intersecting straight lines, constituting an outer and an inner series. But at the present stage I doubt if it is possible to say that this is not accidental.

In connection with the Giant and Dwarf phenomenon and its application to stellar evolution, it should be remembered that all that the fullest survey can give is the instantaneous picture of a system which presumably is changing. The complete picture is a function of many variables, and among these a special value has been given to one—the time. No scrutiny can tell us how to pass from that special value to a general value, which is the goal of stellar evolution. For a given special state different theories will always be possible, explaining its passage to the next state, which *ex hypothesi* is unknown. Obvious as this statement may be, it is not generally seen, and a sweeping postulate of timelessness, or a "steady state" in the observations is called in to mask the deficiency. Such is the basis of the Giant and Dwarf *theory* of development as distinct from a mere record of the phenomenon. It assumes that the observations will show the same features at whatever epoch they are made; therefore each part changes completely and concurrently into an adjacent part. It would follow also that the number of recorded examples of any part was directly proportional to the period of time occupied in passing through that part. It is perhaps unnecessary to dwell upon the Giant and Dwarf theory of evolution with its assumption of constancy of mass, because it is now nearly derelict. But, while one is unwilling to treat a large regularity like the preponderant numbers of great M- and K-stars as a mere accident of distribution, it should be recognised there may be no answer to the question, Why is it there? As suggested on p. 396, it may simply mean that there is a sub-cluster superposed on the main cluster.

The parameters D_i are indicative of the spread of each group. As has been shown, $\frac{3}{2}D_i$ is the mean distance for the group. As the group is not limited outwards, there is no other linear measure to give the size. D_i will stand for it in any questions of statistical locality. For example, we may bring into correlation the known average radial velocity of each spectral class with the value of $\frac{3}{2}D_i$, the mean distance, which belongs to all the groups of that class. As shown in the following table, a remarkable relation emerges.

TABLE III.

Class.	V_R. km./sec.	$2\lambda V_R$.	$\lambda(\frac{3}{2}D_i)$.	$\lambda(V_R^2 \times \frac{3}{2}D_i)$.
B	6·5	1·63	2·36*	3·99
A	10·9	2·07	1·94	4·01
F	14·4	2·32	1·69	4·01
G	15·0	2·35	1·63	3·98
K	16·8	2·45	1·53†	3·98
M	17·1	2·47	·88‡	3·35

* Mean distance derived from the parallaxes without grouping.
† Omitting Giants before K(*3·00*). ‡ Dwarfs only.

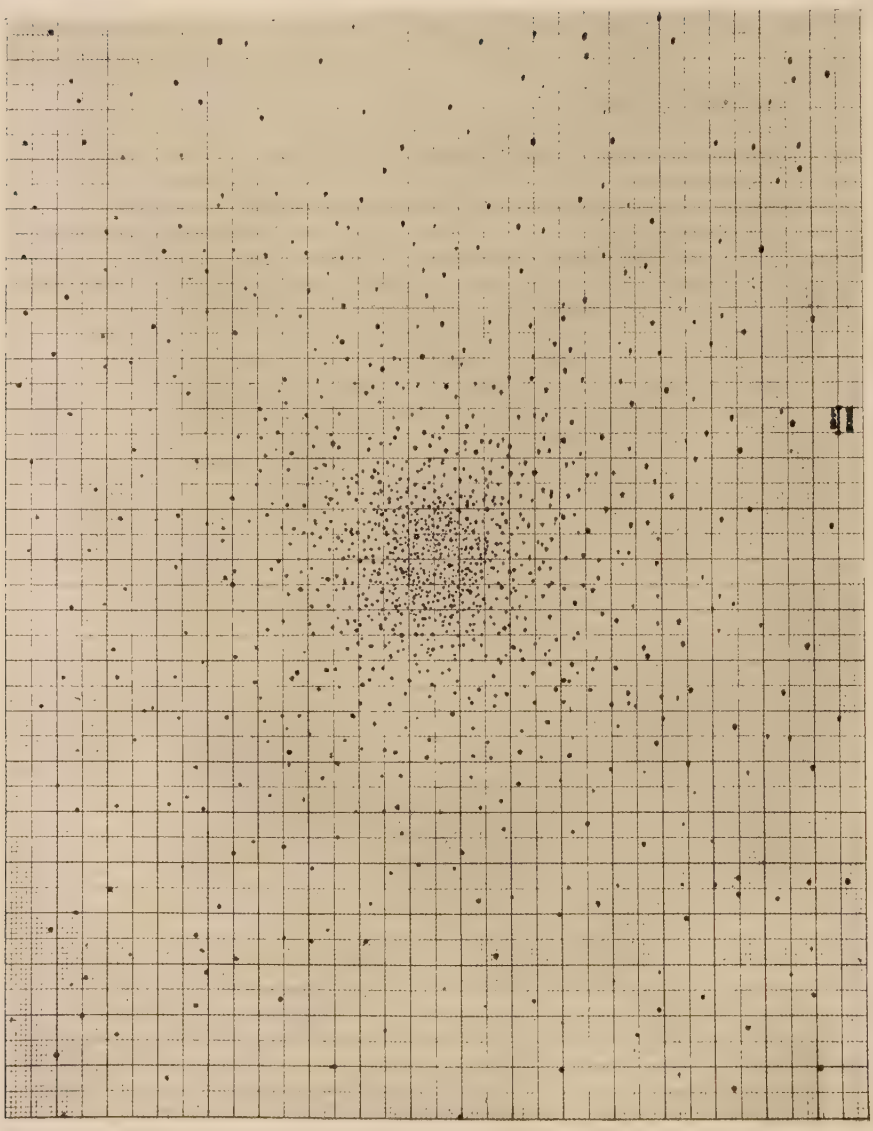
The value of D_i for class M is evidently falsified by the assemblage of different groups together, and by the fact that members are cut off by deficient brightness at the larger distances; otherwise it appears that $V_R^2 \times$ mean distance$=$constant. The procedure, however, is not satisfactory. The value of V^2D should be formed for each star, and the mean of this quantity obtained for each group. Still I cannot believe that the relation is a coincidence. It would be waste of time to fashion in detail a theory that would fit it; but I would express what it implies as follows :—

As Kepler's Third Law, which has the like form, indicates that the motion of the planets is governed by the gravitational field of the Sun, so this indicates that there is a single gravitational field, or distortion of space, due to the presence of mass in the cluster, to which the whole cluster is subject, and that each body moves with the velocity appropriate to its location, of which D_i is indicative.

If we take the analogy from the equation of parabolic motion $v^2r=2\mu$, and measure in terms of the Sun's mass as unit, we find $\mu=1{\cdot}16\times10^6$. The sum of all the masses in our collection is 4067, and if we suppose these are $\frac{1}{10}$th of the whole number down to 8 magnitudes, we should have an outside total of $1{\cdot}6\times10^5$ only. The disparity appears large; but it must be recalled how great would be the effect of numerous, small, non-lucid bodies. If, within the central radius of 10 parsecs, a mass no greater than that of the Moon was found every 600 astronomical units, or within a radius of 1 parsec, dust of the density $1{\cdot}7\times10^{-16}$, the sum would amount in all to $10^6\times$Sun's mass. But a closer investigation must be deferred.

To resume : These charts of distribution constitute a proof that all the proximate stars are arranged as parts of an intricate but clear pattern or system to which the Sun is roughly central—in other words, that a Solar Cluster is a reality. For stars of each temperature a range of masses, from the highest masses downwards, is found, and concurrently, a range of densities, from the most tenuous up to densities many times greater than that of the Sun. We notice that great masses imply small densities, and conversely. But it should be recognised that this rule is merely an implication of the adopted formula connecting mass and luminosity. What is not implied is the actual occurrence of representatives of every group of mass and density in sequence, and their orderly numerical progress.

We must imagine, then, the Sun as marking the centre of the distribution, a typical member of the most numerous group of the most numerous temperature class. Consider any one of the groups, the members of which are the same, roughly, in temperature and mass. They will also be the same in radius, mean density, and luminosity. The members of such a group are distributed in space in concentric shells, which are closest packed at the centre and fall away at a specific rate. Upon each such shell the stars are scattered with a specific density which falls off, as measured by the separation of nearest neighbours, at the same rate as the occurrence of the shells. The number of shells is unlimited and may extend to infinity, but the total number of stars converges to a limit. The rate of falling away is



characteristic of the group. Within once, twice . . . the range of a specific linear parameter, constant proportions are found out of the whole number of stars for any distribution. Therefore, also, it measures the mean distance of members of the group from the centre. This parameter expands regularly from the groups of small mass upwards. It also expands from the classes of low temperature towards those of high temperature. The effect of this is that the B-stars of highest mass are found most widely scattered, while the K- and M-stars of smallest mass are gathered round the centre. But this is only a matter of *frequency* of occurrence. There is nothing to forbid any type of star being found in any locality. Apparently, also, the stars move with velocities appropriate to their distances from the centre.

The evidence is negative of any galactic condensation in the solar cluster. Certainly nothing like the full amount is found. Presumably there is none, and upon the sky the cluster stars are seen superposed to the true galactic stars which manifest the condensation. As the latter are much more remote, they would, on the whole, be fainter, and the superposition of the two systems would explain the progressively greater and greater condensation that is found as fainter and fainter stars are enumerated.

As the cluster lies near the plane of the galaxy we should expect it to be of the open type. To give an idea of its clustered appearance I have plotted the stars considered, correctly as regards distribution and approximate magnitudes, but random in direction. This was done by considering the spheres of radius D_i, $2D_i$ for each group. The proportion of the whole number in the group which is found within, between and beyond, are respectively about $\frac{1}{3}$rd, $\frac{5}{12}$ths, and $\frac{1}{4}$th. These were marked in accordance with an agreed scale of magnitude so as to represent to the best of my judgment a *random* scattering of the few stars in question, each in its appropriate region, as projected upon a plane of vision. The result accords very fairly with the idea of an open cluster. But it must be remembered that the picture is not a correct delineation of the cluster except as to condensation. The globular shape may not be real, since it may have been imparted by the random placing. Further, the picture is far from complete. The series includes many stars of the 8th apparent magnitude; therefore, if it were complete, it would show 40,000 of them. But though the picture is artificial, as far as it goes, the course of the collection suggests to me that the stars shown are a well-balanced representative selection. No suitable open cluster being available, it may be compared with a true globular cluster, such as ω *Centauri* (R.A.S. Photographs, No. 331). Besides the greater richness of the latter, probably more than a hundredfold in density, there is a difference in magnitude of the faintest stars shown, which may amount to 5 magnitudes. The likeness and the contrast of the two figures will be seen at once, and may be left to speak for themselves.

As remarked above, all questions of stellar evolution are essentially theoretical, or even speculative. They are not, for that reason, less fertile. A large amount of information, bounding if not prescribing the course of a permissible theory, may be derived from a study of the adopted Luminosity-Mass-Temperature curves, under the aspects

presented by plotting in relation to one another the various physical quantities which they describe as density, radius, gravitational work, etc. In a certain sense such graphs are results of observation, as being implicit in a curve that agrees with observation, but they cannot be interpreted without a considerable theoretical edifice in addition. It is not my intention to pursue them here, but to keep to the immediate consequences of the observed structures of the groups.

A comment of great interest emerges. Eddington's paper presented the stars as showing a progression in mass, which he interpreted as a progression, generally, in age—a great star dissipating its mass by radiation until it became a small star. Current physical theory requires such a dissipation in order to maintain radiation, but the demonstration of a case where it was going on and could be marked was a new and arresting feature. The structure of the cluster described in the preceding pages puts this into quite a different aspect; it either destroys the evidence for it, or, alternatively, it extends its bearing very widely. It has been shown that the stars are divided into different groups, each group possessing mass, temperature, spatial distribution, and velocity peculiar to itself and different from those of another group. If the difference of mass speaks for a connected time-sequence, it is evident each group as a whole must change into a following group, together with all its characteristics. I would remark that, mathematically, this is quite possible. Each group is a function of two parameters—mass and temperature. Hence the imagined transformation does not assign a law of development, nor does it rule one out. It permits evolution to take place along any family of curves which assigns a relation between M and T, and so is left with one parameter which is available to represent a specific initial state.

The form of such a curve has still to be settled, and it remains to be seen whether one can be assigned that will satisfy all the requirements. But this is a matter with which very able hands have been occupied for some time, and I do not offer any suggestions.

It is relevant, however, to remark that the mass and mean distance appropriate to each group are demonstrably related. The annexed table shows the values of $\lambda(M/\bar{\imath}D_i)$ for all groups. In these, class (F) shows little evidence of progression, classes (G), (K), (M) diminish somewhat as λM diminishes, while class (A) shows an increase. On the whole $\lambda(M/\bar{\imath}D_i)$ changes very little; and if we may carry over the relation of $1/D_i$ with V^2, which was shown to hold for the mean of all the groups of each class to each group individually, this would show that MV^2 throughout a class, and even from class to class, changed little and slowly, while the mass of individual members was reduced to $\frac{1}{10}$th or less. Perhaps class (A) should be treated as an exception. As before, it is not worth while to build up a theory upon this evidence; but it seems clear there is, so far, nothing untenable in supposing that the groups do, in fact, change into one another, condensing upon the centre and increasing in the velocity of their members, as they dissipate their mass. The fact that the groups contain different numbers would explain itself as a measure of the period of time required for the development to pass through any given state. Thus the large numbers found in and before

TABLE IV.

Collected $\lambda(M/\frac{3}{2}D_i)$.

Group.	B.	A.	F.	G.	K.	M.
5·0	..	..	..	..	..	*1·02*
4·5	..	..	..	..	*1·14*	
4·0	..	..	..	..	*1·28*	*1·27*
3·5	..	..	..	..	*1·28*	
3·0	..	..	..	*1·32*	*1·29*	
2·5	..	..	..	*1·39*	*1·39*	
2·0	..	..	*1·48*	*1·49*	*1·45*	
1·5	..	*1·51*	*1·55*	*1·54*	*1·45*	
1·0	..	*1·50*	*1·48*	*1·52*	*1·44*	
0·5	*1·24**	*1·26*	*1·38*	*1·54*	*1·46*	
0·0	..	*1·35*	*1·42*	*1·52*	*1·57*	*1·48*
0·5	..	*1·27*	*1·54*	*1·53*	*1·54*	
1·0	..	..	*1·46*	*1·59*		

the group G (0·0) would indicate a stationary state at that position, while the small numbers at G (0·5) and thereafter would indicate a very rapid movement when that state was passed. But I do not pursue these ideas further.

In conclusion, though the main facts presented above appear to be very clear, the details of the clustering require a great deal of definition. Thus it is desirable to have a complete study of the presence or absence of galactic relations, also a delineation of the appearance of the cluster when each star is represented by a true projection. It is, of course, a familiar fact that the B-stars are restricted to a comparatively narrow region of the sky, and are not found scattered indifferently in all directions. Other eccentricities of distribution may be present, which are not pronounced enough to have gained notice otherwise, and have not declared themselves in this first survey, unless it be in the cases where the fit of the theoretical curve to the plotted observations is a little inferior. Again, investigation should be made of cross motion and radial motion for individual members of the groups in relation with locality. But these and other points I reserve for later treatment. It is hardly too much to say that when the structure of the cluster is fully established, every piece of knowledge we have, whether detailed or statistical, will require to find its place in relation to it, since it has proved that a cross classification according to mass must be made, in addition to the familiar classification according to temperature, before clear and significant relations can be stated.

I have added in Schedule III. a list of the stars of mean density thirty times and upwards that of the Sun. For all these cases the parallax is large, and therefore the calculated particulars are subject to little uncertainty. Though the details are, for the most part, less extraordinary than those of the companion to *Sirius*, they are sufficiently remarkable to be put on record.

* Derived directly without grouping.

Much has been written on the subject of a local cluster by Charlier, Shapley, and others,* but it is unnecessary to deal with these writings in detail, because they contemplate a more extensive system than that here described, namely, one which is related directly with the Milky Way, as well as with other loose clusters about its plane. In the present paper I confine attention to the Solar Cluster alone, which may roughly be envisaged as that collection of stars of which it is possible to make detailed measures of distance and spectrum. I imagine the Solar Cluster is a specimen of a fairly numerous class, so that once we succeed in determining its structure, and perhaps its history, it may become the key to others of its own kind, and beyond.

Professor van Rhijn's paper, "On the Frequency of the Absolute Magnitudes of the Stars" (*Gröningen Publications*, No. 38, 1925), reached me after my discussion was completed. This will explain why I have made no current references in the preceding pages to this learned and comprehensive memoir, which traverses the same field and employs the same material. It is not necessary nor useful to alter what has been written, because the two papers show distinct conceptions and pursue separate paths. Nor have I noticed any place in which their results conflict.† Professor van Rhijn's memoir is constructed strictly on the conceptions of luminosity and spectral class, whereas one of my purposes is to show the advantage of working with mass, density, and temperature, even though they are imperfectly defined. But the most important difference is that he does not assign the distribution in space of any of his groups. He does, indeed, show that a distribution of parallax according to a Gaussian Error Function as used by Kapteyn must be rejected. But no other conception takes its place, and the final tables of his memoir give just the densities of groups "near the Sun," that is at a single point, without introducing the idea of a density falling off with distance and implying clustering. Not, however, that I would be understood to disparage this admirable memoir, the treatment of which many may prefer.

SCHEDULE I.

Class B.

1. Group.	2. No. of Stars.	3. λT.	4. λM.	5. λa.	6. $\lambda \rho$.	7. M.	8. N_u.	9. D_i.	10. D_0.
4·0	2	·43	>2·00	1·93	*<3·79*	*9·06*			
2·5	2	·41	1·43	1 20	*2·32*	*5·48*			
2·0	2	·47	1·35	1·02	*1·85*	*5·16*			
1·5	2	·48	1·17	·83	*1·51*	*4·35*		(all)	
1·0	5	·43	·78	·51	*·94*	*2·25*	19	60	33·2
0·5	6	·47	·65	·28	*·39*	*1·51*			
0·0	3	·46	·46	·06	·14	*·48*			
0·5	1	·41	·24	*·20*	·69	1·52			
1·0	1	·49	·20	*·40*	1·22	1·67			

* *Cf.* Doig, *B.A.A. Journal*, **36**, 116, where full references will be found.

† But notice in any comparison, absolute magnitude in the *Gröningen Publications* corresponds to a distance of 1 parsec and not 10 parsecs.

SCHEDULE I.—*continued.*

Class A.

1. Group.	2. No. of Stars.	3. λT.	4. λM.	5. λ*a*.	6. λρ.	7. M.	8 N_u.	9. D_i.	10. D_0.
2·0	7	·30	·93	·90	*1·88*	*2·86*			
1·5	14	·32	·75	·66	*1·46*	*1·86*	22	120	63·0
1·0	35	·31	·58	·46	*·95*	*·76*	46	80	32·6
0·5	41	·30	·42	·24	*·47*	·36	40	32	13·7
0·0	27	·31	·31	·06	*·04*	1·23	26	30	15·1
0·5	13	·28	·16	*·16*	·53	2·62	16	18	10·6
1·0	6	·30	·06	*·36*	·97	3·38			
1·5	6	·33	·01	*·52*	1·45	3·88			
2·0	2	·29	*·15*	*·76*	1·95	5·47			
5·0	1	·33	*·65*	*1·89*	4·87	10·75			

Class F.

4·5	1	·08	1·85	2·08	*4·54*	*6·59*			
3·5	1	·16	1·56	1·70	*3·72*	*5·49*			
3·0	1	·16	1·31	1·48	*3·11*	*4·39*			
2·5	9	·14	·82	1·02	*2·49*	*1·88*			
2·0	13	·12	·60	·80	*1·97*	*·62*	13	80	55·7
1·5	21	·10	·41	·56	*1·44*	·81	21	60	33·2
1·0	30	·12	·33	·39	*·98*	1·44	31	43	20·1
0·5	42	·11	·18	·16	*·46*	2·69	42	24	10·0
0·0	57	·12	·10	*·01*	*·05*	3·44	58	22	8·3
0·5	29	·12	*·02*	*·22*	·48	4·48	37	22	9·7
1·0	9	·15	*·10*	*·43*	1·01	5·26	12	15	10·5
1·5	1	·08	*·27*	*·66*	1·66	7·11			
2·0	1	·08	*·38*	*·90*	2·17	8·31			
3·0	1	·16	*·44*	*1·16*	2·86	8·81			
6·0	1	·16	*·98*	*2·39*	6·19	15·26			

Class G.

5·5	1	*·08*	1·87	2·36	*5·36*	*6·38*			
4·5	2	*·04*	1·48	1·94	*4·48*	*4·62*			
4·0	3	*·05*	1·08	1·60	*3·88*	*2·87*			
3·5	8	*·05*	·85	1·36	*3·40*	*1·71*			
3·0	14	*·06*	·67	1·16	*2·95*	*·62*	14	65	39·6
2·5	19	*·04*	·53	·95	*2·49*	·25	15	55	32·3
2·0	21	*·06*	·36	·73	*1·97*	1·55	19	47	26·0
1·5	34	*·03*	·25	·51	*1·48*	2·35	29	40	19·0
1·0	70	*·03*	·14	·31	*·97*	3·35	79	40	13·6
0·5	79	*·02*	·04	·13	*·47*	4·17	80	25	8·5
0·0	101	*·02*	*·06*	*·10*	*·02*	5·30	100	19	6·0
0·5	36	*·02*	*·15*	*·26*	·45	6·06	34	16	7·2
1·0	15	*·01*	*·23*	*·44*	·98	6·92	19	15	8·3
1·5	6	·00	*·27*	*·58*	1·42	7·48			
2·0	1	·00	*·38*	*·77*	1·78	8·45			

SCHEDULE I.—*continued.*

Class K.

1. Group.	2. No. of Stars.	3. λT.	4. λM.	5. λα.	6. λρ.	7. M.	8. N_u.	9. D_i.	10. D_0.
6·5	1	*·16*	>2·18	2·89	<*6·49*	*7·95*			
6·0	3	*·22*	1·83	2·55	*5·94*	*5·94*			
5·5	1	*·25*	1·58	2·38	*5·68*	*4·79*			
5·0	5	*·20*	1·35	2·10	*5·08*	*3·92*			
4·5	17	*·20*	1·04	1·79	*4·50*	*2·36*	25	100	50·3
4·0	27	*·18*	·85	1·56	*3·97*	*1·40*	31	90	42·0
3·5	41	*·17*	·64	1·34	*3·47*	*·39*	47	55	22·4
3·0	35	*·17*	·49	1·10	*2·98*	·78	37	40	17·6
2·5	29	*·17*	·35	·89	*2·50*	1·85	31	36	16·8
2·0	19	*·18*	·21	·68	*1·98*	3·01	19	30	16·6
1·5	22	*·17*	·11	·49	*1·48*	3·84	21	24	12·6
1·0	45	*·18*	*·01*	·26	*·97*	5·09	49	18	7·2
0·5	44	*·17*	*·10*	·08	*·51*	5·88	49	15	6·0
0·0	24	*·19*	*·21*	*·11*	*·09*	7·05	36	15	6·6
0·5	13	*·20*	*·32*	*·32*	·50	8·20	22	11	5·8
1·0	5	*·21*	*·41*	*·50*	1·10	9·23			
1·5	2	*·25*	*·55*	*·72*	1·51	10·74			
2·0	1	*·16*	*·57*	*·92*	2·07	10·80			

Class M.

6·5	1	*·26*	2·23	2·92	*6·68*	*7·41*			
6·0	1	*·30*	1·70	2·56	*5·80*	*5·22*			
5·5	3	*·26*	1·43	2·25	*5·46*	*4·04*			
5·0	10	*·27*	1·14	2·00	*5·02*	*2·69*	16	100	58·8
4·5	6	*·27*	·91	1·78	*4·53*	*1·59*			
4·0	13	*·27*	·69	1·54	*4·03*	*·42*	16	60	35·3
3·5	3	*·27*	·47	1·30	*3·57*	·78			
3·0	3	*·26*	·40	1·12	*3·10*	1·61			
2·5	1	*·28*	·29	·96	*2·74*	2·60			
2·0	0								
1·5	2	*·29*	*·04*	·46	*1·55*	5·22			
1·0	0								
0·5	5	*·27*	*·22*	·04	*·41*	7·10			
0·0	8	*·27*	*·29*	*·15*	·03	8·05	31	10	4·7
0·5	5	*·26*	*·39*	*·34*	·40	8 91			
1·0	2	*·27*	*·48*	*·54*	1·01	10·02			
1·5	1	*·28*	*·56*	*·71*	1·42	10·95			
2·0	1	*·28*	*·65*	*·92*	1·96	12·00			

SCHEDULE II.

Group G (0·0).

No.	π.	M.	λT.	λM.	λα.	λρ.
	″					
1	·018	6·62	·03	*·21*	*·02*	*·15*
2	·033	6·40	·00	*·18*	*·11*	·15
3	·050	6·32	*·08*	*·17*	*·13*	·22
4	·054	6·37	*·08*	*·17*	*·14*	*·25*
5	·043	6·37	*·08*	*·17*	*·14*	*·25*
6	·079	6·12	*·08*	*·17*	*·09*	·10
7	·052	6·32	*·08*	*·17*	*·13*	·22
8	·031	6·27	*·08*	*·16*	*·08*	·20
9	·029	6·12	*·08*	*·15*	*·09*	·12
10	·037	6·17	*·08*	*·15*	*·10*	·15
11	·043	6·17	*·08*	*·15*	*·10*	·15
12	·036	6·02	*·08*	*·14*	*·07*	·07
13	·034	6·07	*·08*	*·14*	*·08*	·10
14	·200	5·92	*·08*	*·14*	*·05*	·01
15	·046	6·02	*·08*	*·14*	*·07*	·07
16	·143	5·92	*·08*	*·13*	*·05*	·02
17	·056	5·97	*·08*	*·13*	*·06*	·05
18	·052	5·87	*·10*	*·12*	·00	*·12*
19	·035	5·82	*·08*	*·12*	*·03*	*·03*
20	·023	5·70	·00	*·12*	·03	*·21*
21	·027	5·77	*·08*	*·11*	*·02*	*·05*
22	·054	5·77	*·08*	*·11*	*·02*	*·05*
23	·026	5·77	*·08*	*·11*	*·02*	*·05*
24	·022	5·72	*·08*	*·11*	*·01*	*·08*
25	·194	5·77	*·08*	*·11*	*·02*	*·05*
26	·037	5·77	*·08*	*·11*	*·02*	*·05*
27	·041	5·67	*·08*	*·10*	·00	*·10*
28	·124	5·57	*·08*	*·09*	·02	*·15*
29	·022	5·40	·00	*·09*	*·11*	·24
30	·049	5·35	·00	*·09*	*·10*	·21
31	·030	5·52	*·08*	*·09*	*·18*	·00
32	·030	5·40	·00	*·09*	*·11*	·24
33	·035	5·40	·00	*·09*	*·11*	·24
34	·032	5·57	*·08*	*·09*	·02	*·15*
35	·051	5·57	*·08*	*·09*	·02	*·15*
36	·032	5·57	*·08*	*·09*	·02	*·15*
37	·042	5·52	*·08*	*·09*	·03	*·18*
38	·018	5·42	*·08*	*·08*	·05	*·23*
39	·061	5·25	·00	*·08*	*·08*	·16
40	·031	5·47	*·08*	*·08*	·04	*·20*
41	·031	5·47	*·08*	*·08*	·04	*·20*

SCHEDULE II.—*continued.*

No.	π.	M.	λT.	λM.	λa.	λρ.
42	″·026	5·47	*·08*	*·08*	·04	*·20*
43	·038	5·42	*·08*	*·08*	·05	*·23*
44	·040	5·42	*·08*	*·08*	·05	*·23*
45	·017	5·15	·02	*·07*	*·09*	·20
46	·048	5·20	·00	*·07*	*·07*	·14
47	·065	5·15	·00	*·07*	*·06*	·11
48	·140	5·05	·00	*·06*	*·04*	·06
49	·042	5·10	·00	*·06*	*·05*	·09
50	·043	5·05	·00	*·05*	*·04*	·07
51	·079	5·10	·00	*·06*	*·05*	·09
52	·057	5·10	·00	*·06*	*·05*	·09
53	·092	4·92	·03	*·05*	*·06*	·13
54	·043	5·05	·00	*·05*	*·04*	·07
55	·040	4·95	·00	*·05*	*·02*	·01
56	·050	4·92	·03	*·05*	*·08*	·19
57	·039	4·95	·00	*·05*	*·02*	·01
58	·088	5·00	·00	*·05*	*·03*	·04
59	·057	4·82	·03	*·04*	*·06*	·14
60	·120	4·80	·00	*·02*	·01	*·05*
61	·056	4·75	·00	*·02*	·02	*·08*
62	·029	4·62	·07	*·02*	*·08*	·22
63	·069	4·80	·00	*·02*	·01	*·05*
64	·023	4·70	·00	*·01*	·03	*·10*
65	·079	4·70	·00	*·01*	·03	*·10*
66	·048	4·62	·03	*·01*	*·02*	·05
67	·081	4·67	·03	*·01*	*·01*	·02
68	·031	4·65	·00	*·01*	·04	*·13*
69	·036	4·62	·03	*·01*	*·02*	·05
70	·148	4·67	·03	*·01*	*·03*	·08
71	·101	4·62	·03	*·01*	*·02*	·05
72	·038	4·70	·00	*·01*	·03	*·10*
73	·030	4·62	·03	*·01*	*·02*	·05
74	·059	4·52	*·03*	·00	·00	·00
75	·044	4·58	*·02*	·00	·08	*·24*
76	·054	4·55	·00	·00	·06	*·18*
77	·068	4·55	·00	·00	·06	*·18*
78	·058	4·60	·00	·00	·05	*·15*
79	·037	4·55	·00	·00	·06	*·18*
80	·022	4·60	·00	·00	·05	*·15*
81	·032	4·65	·00	·00	·04	*·12*
82	·035	4·60	·00	·00	·05	*·15*
83	·035	4·60	·00	·00	·05	*·15*

SCHEDULE II.—*continued.*

No.	π.	M.	λT.	λM.	λa.	λρ.
	″					
84	·043	4·45	·00	·01	·08	*·23*
85	·036	4·45	·00	·01	·08	*·23*
86	·028	4·47	·03	·01	·01	*·02*
87	·076	4·50	·00	·01	·07	*·20*
88	·012	4·42	·03	·01	·02	*·05*
89	·026	4·45	·00	·01	·08	*·23*
90	·015	4·50	·00	·01	·07	*·20*
91	·025	4·42	·03	·01	·02	*·05*
92	·022	4·50	·00	·01	·07	*·20*
93	·035	4·42	·03	·01	·02	*·05*
94	·011	4·50	·00	·01	·07	*·20*
95	·018	4·32	·03	·02	·04	*·10*
96	·013	4·27	·03	·03	·05	*·12*
97	·025	4·12	·03	·04	·08	*·20*
98	·046	4·12	·03	·04	·08	*·20*
99	·016	4·20	·00	·04	·05	*·11*
100	·054	4·05	·00	·05	*·04*	·17
101	·013	4·07	·03	·05	·09	*·22*
Means						
	·049	5·30	*·02*	*·06*	*·10*	*·02*

SCHEDULE III.

Stars of High Density.

1. Class.	2. R.A.	3. Dec.	4. π.	5. *m.*	6. M.	7. λM.	8. λa.	9. λρ.
	h m	° ′	″					
A	16 6	−19 12	·042	6·5	4·00	*·01*	*·49*	1·46
A	21 18	+ 6 23	·079	5·1	4·00	*·01*	*·49*	1·46
A	18 45·0	+49 19	·012	9·3	4·10	*·02*	*·51*	1·51
Ao	3 40·5	+23 57	·030	7·0	3·80	·01	*·55*	1·66
A5	13 28·3	+35 25	·067	6·8	5·65	*·16*	*·65*	1·79
Ao	3 41·4	23 50	·040	8·1	5·28	*·14*	*·75*	2·11
A	4 10·8	− 7 48	·214	9·7	10·75	*·65*	*1·84*	4·87
F5	11 52	− 0 54	·053	8·5	7·11	*·27*	*·61*	1·66
F5	9 45·5	−11 48	·095	8·4	8·31	*·38*	*·85*	2·17
Fo	16 42·5	+33 59	·143	8·1	8·81	*·44*	*1·10*	2·86
Fo(M.W.)	0 43·9	+ 4 55	·244	12·3	15·26	*·98*	*2·39*	6·19
G5	13 40	+18 24	·077	9·6	8·87	*·40*	*·64*	1·52
Go	15 4·8	−15 54	·064	9·4	8·45	*·38*	*·72*	1·78
K5	18 41·8	+59 29	·293	9·3	10·56	*·53*	*·65*	1·42
K5	18 41·8	+59 29	·278	9·8	10·91	*·56*	*·72*	1·60
Ko	0. 43·0	+57 17	·178	7·6	8·30	*·34*	*·88*	2·20
Mb	22 24	+57 11	·254	9·3	10·95	*·56*	*·66*	1·42
Mb	17 52·9	+ 4 28	·538	9·7	12·00	*·65*	*·87*	1·96

Addendum, 1926 April 14.

It is the wish of the Council that this paper should appear together with certain objections raised by the Referees and my replies to them.

A. Consider a group of N stars showing the same parallax π_0. The number with observed parallaxes π to $\pi+d\pi$ is

$$N\frac{h}{\sqrt{c}}e^{-h^2(\pi-\pi_0)^2}d\pi \qquad [c=3\cdot1416\ldots$$

Equating this to $4cr^2\rho dr=4c\rho d\pi/\pi^4$, so that ρ is the corresponding star-density, we have

$$\rho=\pi^4e^{-h^2(\pi-\pi_0)^2}\times const.$$

This is a maximum when

$$2h^{-2}=\pi(\pi-\pi_0)$$

or

$$4\mu^2=\pi(\pi-\pi_0)$$

where μ is the mean square error of the observed parallaxes. For the best parallaxes μ is about $''\cdot01$, so that if

$\pi_0=''\cdot00$, ρ_{max} is at 50 parsecs,
$\pi_0=''\cdot01$, ρ_{max} is near 40 parsecs;

so that all the more distant stars *appear* to cluster near 40–50 parsecs.

This interesting point was overlooked, and I am indebted to the Referees for raising it. It amounts to saying that by the operation of accidental error a finite number of the more distant stars will be drafted into the nearer, and therefore narrower zones, so that the occurrence of maximum density may be totally displaced. Account must, however, be taken of the fact that the redistribution concerns all zones, and that N is a function of π_0. Stated more comprehensively, the problem is the following: *To find the relation between the apparent star-density* ρ *and the true star-density* P, *when the parallaxes are subject to an assigned accidental error.*

Say D is the distance, and $D\pi=1$. The number of stars from D to $D+dD$ is

$$4c\mathrm{P}D^2dD=-4c\mathrm{P}\pi^{-4}d\pi.$$

The proportion of these found between π' and $\pi'+d\pi'$ is

$$c^{-\frac{1}{2}}he^{-h^2(\pi-\pi')^2}d\pi',$$

and these contribute an element $d\rho'$ to the star-density observed there, so that

$$d\rho'=c^{-\frac{1}{2}}hd\pi\cdot e^{-h^2(\pi-\pi')^2}\cdot\mathrm{P}(\pi'/\pi)^4.$$

The limits of integration are

$$\pi\Big|_0^\infty$$

This is the expression required.

An important conclusion may be drawn from it immediately. For ρ' to converge to a finite limit it is necessary that

$$\int_0 d\pi \mathrm{P}\pi^{-4} = \int^{\infty} \mathrm{P}\mathrm{D}^2 d\mathrm{D}$$

should converge—that is, the total number of stars up to infinity must be finite. If a parallax list were made by a random selection from an unlimited universe of stars, the more distant members would stultify it by their intrusion into every other zone through the operation of accidental error, and would swamp the stars proper to that zone. Hence a parallax list must of necessity relate to a limited group.* Such a limited group may be either a natural cluster, as I maintain it to be, or it may be artificially produced by a process of selection, as, *e.g.*, by interposing a boundary zone between the parallax stars and the comparison stars of the background. I return to this point later. Consider now the expression for $d\rho'$. Put

$$\mathrm{P} = \kappa\pi^4 ;$$

then

$$d\rho'/\mathrm{P}' = c^{-\frac{1}{2}} h d\pi \cdot e^{-h^2(\pi-\pi')^2} \cdot (\kappa/\kappa').$$

Hence, if the form of κ were known the elements $d\rho'$ could be summed by drawing ordinates of the error curve

$$\mathrm{E} = c^{-\frac{1}{2}} h e^{-h^2(\pi-\pi')^2},$$

and finding the area of the new curve with ordinates

$$\mathrm{F} = \mathrm{E}\kappa/\kappa'.$$

The value of ρ' is then

$$\mathrm{P}' \int_0^{\infty} \mathrm{F} d\pi.$$

Strictly speaking, ρ' is the datum from observation and P′ what is required. In order, however, to investigate the question in the simplest way I enquire what would be the actual value of ρ' if P′ had the form $Ae^{-k\mathrm{D}}$ deduced in the foregoing pages. Taking the special case of the Group G (0·0) I find the following results by using the method indicated above, and employing mechanical integration—the accent is omitted as having no further significance—

Group G (0·0).

π.	ρ/P.	P/P_0.	ρ/P_0.
″			
·10	·95	·45	·43
·05	1·11	·20	·22
·04	1·08	·14	·15
·03	·90	·07	·06
·02	·73	·02	·01
·01	1·33	·00	·00

* It is a very important conclusion that this condemns the unselected employment of all the stars in a field for parallax work, which is a part of Kapteyn's Plan of Selected Areas.

These changes have no significance; the conclusion is the same whether we employ P or ρ; and it does not seem necessary to examine all the other groups.

Briefly one may say that the redistribution of parallaxes by accidental error hardly operates before parallax ″·01, and it is there ineffective because there are so few stars to redistribute.

One further ramification of the argument may be indicated. A parallax affected by error affects in turn the luminosity, mass, and density. Hence the occurrence of accidental error may transfer stars from one group to another. The groups, however, are fairly broad, and there does not appear to me to be any risk that this would occur to any degree requiring attention.

B (1). A Solar Cluster implies stars near the Sun and moving with the Sun. A large proportion of the stars whose parallaxes are known have considerable proper motions. A proper motion of as much as 20″ a century, will in a million years account for a transverse movement equal to the star's present distance. There is no permanent association of the stars we know to be near the Sun, with it. A star of considerable parallax and small proper motion is hardly known.

We must distinguish between what is known and what is mere construction, though currently accepted as known. We have no knowledge whatever of the configuration of the stars a million years ago or a million years hence, or whether there is permanent association between the Sun and other stars. Two things only are possible to learn—(1) the instantaneous space configuration, and (2) the instantaneous velocity system. The former is determined in the foregoing pages. It cannot be upset except by changing the material on which it is based, proving it to be wrong, inadequate, or unrepresentative. In the meantime it is the whole of the material we have got. It shows what I describe as a cluster of definite structure. I say nothing as to its permanence, though I should guess it to be permanent and not a transient congress. But if it is to be permanent the stars must move in orbits about the centre, and not in straight lines. The collection of matter which this implies is briefly treated in p. 400; but as there stated, the question of velocities requires an independent discussion. A statistical treatment of the velocity diagram might show regularities as clear as those exhibited by the space diagram. It is possible that relations would emerge between the two, such as that adumbrated on p. 400. But no consideration of motions can upset the ascertained space configuration.

B (2). The rejection of negative parallaxes is open to grave objection. They amount to nearly 15 per cent. of the total. It is quite possible that useful results might be obtained by considering only the stars whose parallaxes are greater than ″·03 or ″·04, as the probable error is a little less than ±″·01; but it is difficult to see how any results can be based on the parallaxes between ″·01 and ″·00 when the negative results are omitted.

There are two obvious causes that may give rise to negative parallaxes: (1) The intrusion of accidental error greater than the actual parallax with a negative sign. Under my reply to (*A*) above, the effect of accidental error over the whole of the rest of its range is considered and shown to be unimportant statistically. The limits of integration treated in that reply are in effect the same thing as the rejection of negative parallaxes arising from this cause. No more need, therefore, be said on this point. (2) A negative parallax may also occur by taking as parallax star one that is really a member of the background. That this occurs so often shows, as remarked on p. 398, that the selection rules adopted, whatever they may be, are largely ineffective, and that the stars are more completely a random list than was contemplated. Where a negative parallax is due to this cause, it is evidently the proper course to reject it—in other words, to treat the observation as not made, as one would do if it contained any other known mistake.

If it is desired to see what results appear by confining attention to the first 30 or 40 parsecs, that can be done by inspection of Plate 3.

The characteristic curves of distribution assert themselves quite clearly in all the groups of higher density, together with their progression, while in the other groups they show in a truncated form. This is because 40 parsecs is not sufficient to span the cluster—a radius of 100 parsecs is required; and it is assumed for trial, and verified by trial that down to this distance the collection may be treated as homogeneous in quality. No results whatever are based on parallaxes between ″·01 and ″·00.

B (3). The result on which great stress is laid is the point of inflection in the curves, *i.e.* the *second* differences. It seems difficult to believe that *second* differences obtained in this way are due to a cosmical fact and not to the method of treatment.

There is no conceivable way in which clustering can be demonstrated mathematically except to show that the rate of enumeration by zones from the centre, starting at zero, mounts to a maximum and falls to zero again. This, in plain language, is the "second difference" objected to. The method of treatment was deliberately chosen to be as featureless as possible in order that it might be visible that there was no loophole by which the regularities detected might have been introduced. It will be noted that nothing is alleged against it except a difficulty to believe. But a cosmical fact may be as well expressed by second differences as by any other mathematical device. The acceleration of matter is a cosmical fact; it is presented directly to the senses as a measure of force, but for mathematical purposes it is preferred to describe it as a second difference. But if anyone regards the appeal to second differences as too abstruse to build belief upon, he may be referred to Plate 5, where exactly the same facts of progressive clustering are presented directly to the senses in the form of a picture.

B (4). The lists of stars observed for parallax have been with few exceptions based on magnitude of proper motion, or on apparent brightness. Thus the material is selected in a complicated manner, and discussion without recognition of this fact is liable to give erroneous results.

In one respect only it may be admitted that the selections made by parallax observers touch the conclusions of this paper. Knowing that the probable error will not fall much below ″·01, it may be supposed that stars would be generally excluded for which the anticipated parallax was less than this amount. Moderately reliable anticipations are derivable from the star's proper motion, though the presence of negative parallaxes in large numbers shows that the rule cannot be trusted. It may be argued that this constitutes the introduction of a boundary and the creation of an artificial cluster, such as I refer to on p. 411 above. I should admit this. But that is a very small part of the structure described in my paper, which runs in a connected sequence through all the spectral classes and stars of all densities within them. Parallax observers have had no idea that they were describing or helping to describe such a structure. From the point of view of this structure the material they offered, derived from forty-four publications by six observatories, is fully random. If, however, anyone prefers to reject the picture that is presented on analysing this material, because he presumes that it is liable to some unassigned error, I have nothing to say. My paper is addressed to others.

Precession, Galactic Rotation, and Equinox Correction. By J. K. Fotheringham, D.Litt.

Precession and Galactic Rotation.

In 1910 Lewis Boss, immediately after the publication of his *Preliminary General Catalogue of* 6188 *Stars*, published in the *Astronomical Journal*, xxvi, 95–99, 111–122, two papers dealing with precession and solar motion. Boss's discussion is distinguished from all its predecessors by the fact that, largely through his own labours, he was able to include the whole heavens in his survey. He divided the sky into 100 trapeziums of approximately equal area, and 16 half-sectors, each approximately half the size of one of his trapeziums. He introduced into his discussion 5413 stars having a mean magnitude on the scale which he used of 5·7, equivalent to about 5·5 on the Harvard scale. The number of stars in each trapezium varied from 24 to 107 and in each half-sector from 14 to 32. Equations of condition were formed for each star and a mean was found of all the equations in each trapezium or half-sector. These mean equations were again treated as equations of condition in a least squares solution in which all the trapeziums received equal weight and each half-sector received half the weight of a trapezium. In this way the effect of systematic motions has probably been eliminated as successfully as is at present possible.

In forming his equations Boss made an important departure from the established practice. Adopting Newcomb's precession and the equinox of Newcomb's system N_1, he used Δk for the correction to be applied to the mean motion in right ascension of all the stars measured

from the assumed equinox,* and Δn for the correction to the motion of the pole of the equator round the pole of the ecliptic, expressed as an arc of a great circle; the three usual solar motion terms were also introduced :—

$$X=M \cos D \cos A$$
$$Y=M \cos D \sin A$$
$$Z=M \sin D,$$

where M is the centennial parallactic drift at 90° from the solar apex, A is the right ascension and D the declination of the solar apex at epoch 1875. He thus obtained equations in right ascension of the form :—

$$\Delta k \cos \delta+\Delta n \sin \alpha \sin \delta+X \sin \alpha-Y \cos \alpha=\mu \cos \delta,$$

and in declination of the form :—

$$\Delta n \cos \alpha+X \cos \alpha \sin \delta+Y \sin \alpha \sin \delta-Z \cos \delta=\mu'.$$

It had previously been the practice to use equations of the form

$$\Delta p (\sin \alpha \sin \delta \sin \epsilon+\cos \delta \cos \epsilon)+X \sin \alpha-Y \cos \alpha=\mu \cos \delta$$
$$\Delta p \cos \alpha \sin \epsilon+X \cos \alpha \sin \delta+Y \sin \alpha \sin \delta-Z \cos \delta=\mu',$$

where Δp is the correction to the assumed value of the general precession in longitude and ϵ is the obliquity of the ecliptic.

The advantage of the method used by Boss is that it enables us to distinguish Δk, which includes equinox correction and any correction that may be necessary to precession on the equator, from $\Delta n \cot \epsilon$ which includes only precession on the ecliptic, and it also permits us to determine the precession on the ecliptic from right ascension and declination independently. But even in Boss's form the equations do not exhaust the geometrically possible secular motions of the equinoxes in relation to the mean of the stars. They represent by separate unknowns a rotation of the stellar system on the plane of the ecliptic, and such a rotation on the plane of the equator. There remains the possibility of a rotation on some other plane. The idea of a rotation on the plane of the galaxy has long been familiar to astronomers, and Poincaré † has computed that the rotation ought to be at the rate of $\frac{1}{5}$ of a second of arc per century, but he concludes that this rotation cannot be observed, because in order to determine it by observation it would be necessary to compare it with nebulæ lying outside the galaxy and not participating in its motion. This seems to assume that the galactic rotation will carry with it the inertial frame of the stellar system and the equatorial plane, a very doubtful assumption. At all events it is quite simple to modify the equations of condition so as to introduce such a rotation on the assumption that it does not carry the equatorial plane with it. This was done by Ludwig Struve in his treatise of 1887 on the determination of the constant of precession.‡ Struve cites similar studies by Rancken

* In his *Preliminary General Catalogue*, Boss had reckoned Δk in the opposite direction.

† *Leçons sur les hypothèses cosmogoniques* (1911), 263.

‡ *Mémoires de l'Académie impériale des sciences de St. Pétersbourg*, vii^e^. série, xxxv., No. 3.

and Bolte. Putting u for the centennial motion of the pole of the equator in relation to the mean of the stars, attributable to the rotation in question, Ω for the right ascension of the ascending node of the plane of rotation, and $\Delta n'$ for $u \cos \Omega + \Delta n$, we have the following equations :—

$$\Delta k \cos \delta + \Delta n' \sin \alpha \sin \delta - u \sin \Omega \cos \alpha \sin \delta + X \sin \alpha - Y \cos \alpha = \mu \cos \delta$$
$$\Delta n' \cos \alpha + u \sin \Omega \sin \alpha + X \cos \alpha \sin \delta + Y \sin \alpha \sin \delta - Z \cos \delta = \mu'.$$

In determining the new term $u \sin \Omega$, I have started with the residuals, which Boss calls C−O, which still remain after he has introduced the solar motion terms and corrections to precessional terms resulting from the solution of the normal equations given on p. 112 of his second paper. From these I have formed new terms in Boss's equations of condition and new normal equations, the equations for determining $u \sin \Omega$ taking the form :—

From Right Ascensions.

$$-\cdot 30 \Delta_2 k - \cdot 31 \Delta_2 n' + 18 \cdot 625\, u \sin \Omega + \cdot 17 \Delta X - \cdot 41 \Delta Y = +2''\cdot 18$$

From Declinations.

$$-\cdot 25 \Delta_2 n' + 53 \cdot 91 u \sin \Omega - \cdot 17 \Delta X + \cdot 35 \Delta Y - \cdot 11 \Delta Z = +8''\cdot 81$$

From Right Ascensions and Declinations Combined.

$$-\cdot 30 \Delta_2 k - \cdot 56 \Delta_2 n' + 72 \cdot 535 u \sin \Omega - \cdot 06 \Delta Y - \cdot 11 \Delta Z = +10''\cdot 99$$

where $\Delta_2 k$, ΔX, ΔY, ΔZ, represent any corrections which it may be necessary to apply to the quantities Δk, X, Y, Z, already found by Boss, and $\Delta_2 n'$ represents the correction to be applied to Boss's Δn to obtain $\Delta n'$. As each quantity is found by Boss with a probable error of $\pm''\cdot 06$, it will be observed that the effect of these further corrections is negligible. My equations differ from Boss's in that I have formed one equation of condition in right ascension and one in declination from the mean right ascension and declination of each trapezium and half-sector, whereas he has meaned the separate equations from the different stars. I regard this difference in method as unimportant. My coefficients of $\Delta_2 k$, $\Delta_2 n'$, X, Y, Z, are taken from terms in Boss's equations which ought to have the same coefficients.

From the above equations I obtain for $u \sin \Omega$ the following values.

From right ascensions . . .	$+0''\cdot 117 \pm ''\cdot 116$
From declinations . . .	$+0''\cdot 163 \pm ''\cdot 065$

From right ascensions and declinations combined $+0''\cdot 152 \pm ''\cdot 057$.

If we ignore solar motion, the solution from right ascensions becomes $+0''\cdot 05$ and that from declinations $+0''\cdot 19$. Solar motion does not affect the combined solution.

Ludwig Struve found for the same term

From right ascensions . . .	$-0''\cdot 037 \pm ''\cdot 683$
From declinations . . .	$+0''\cdot 408 \pm ''\cdot 264$

From right ascensions and declinations combined $+0''\cdot 350 \pm ''\cdot 246$.

It will be observed that the value just found is equal to $2\frac{2}{3}$ times its probable error, and differs from Struve's determination by less than Struve's probable error. In view of the large probable error attaching to his result, and in view of the discordance between his result and Rancken's, Struve decided not to introduce such a term. If I have decided otherwise, it is because the observational material provided by Boss permits a more precise determination.

Following the example set by Boss in respect of the other precessional and solar motion terms, I have determined this term also for different zones of declination and of galactic latitude.

For zones of declination I find

Zone I, $+10°$ to $-10°$	$+0''\cdot38\pm''\cdot16$
Zone II, $\pm10°$ to $\pm30°$	$+0''\cdot18\pm''\cdot12$
Zone III, $\pm30°$ to $\pm50°$	$-0''\cdot10\pm''\cdot11$
Zone IV, $\pm50°$ to $\pm90°$	$+0''\cdot25\pm''\cdot11$

The probable error derived from the discordances of these solutions is $\pm''\cdot067$, or very little more than we should expect if the computed rotation was uniform throughout the heavens.

I have formed zones of galactic latitude as follows, each trapezium being assigned to the zone in which the mean position of its stars lies.

	Trapeziums included.	Value of $u \sin \Omega$
Zone I, $+7°$ to $-7°$	6, 15, 23, 33, 42, 50, 57, 66, 73, 78, 83, 84, 91, 95, 96, 97	$+0''\cdot18\pm''\cdot12$
Zone II, $\pm7°$ to $\pm19°$	5, 7, 14, 16, 22, 24, 32, 34, 41, 43, 49, 51, 56, 58, 67, 68, 74, 75; 77, 79, 90, 98, 101, 104_2, 106_2, 107_1, 107_2	$+0''\cdot09\pm''\cdot078$
Zone III, $\pm19°$ to $\pm42°$	4, 8, 13, 17, 20, 21, 25, 31, 35, 36, 40, 44, 45, 47, 48, 52, 55, 59, 65, 72, 76, 80, 85, 89, 94, 99, 101_2, 102_1, 102_2, 103_1, 103_2, 104_1, 105_1, 105_2, 106_1, 108_1, 108_2	$+0''\cdot25\pm''\cdot11$
Zone IV, $\pm42°$ to $\pm90°$	1, 2, 3, 9, 10, 11, 12, 18, 19, 26, 27, 28, 29, 30, 37, 38, 39, 46, 53, 54, 60, 61, 62, 63, 64, 69, 70, 71, 81, 82, 86, 87, 88, 92, 93, 100	$+0''\cdot07\pm''\cdot15$

The value of $u \sin \Omega$ resulting from these separate solutions is $+0''\cdot14\pm''\cdot053$, if the probable error is derived from the probable errors of the individual solutions, or $+0''\cdot14\pm''\cdot027$ if it is derived from the discordances of the four solutions. It will be observed that all the galactic zone solutions agree with the mean value within their probable errors. It would appear, therefore, that the rotation in question not only exists, but, so far as our observations can test it, has the same value for each galactic zone. If we divide each of Zones II, III, IV into a positive and negative half, so that opposite regions of the sky are no longer united in the same solution, the probable error derived from the discordances of the seven zones amounts to $\pm''\cdot066$, a value not greatly in excess of that obtained from the discordances of the individual trapeziums, but contrasting sharply with the remarkable accord shown

by the four galactic zones. If this contrast is not purely accidental, it would seem to imply that the elimination of solar motion, which is probably only a resultant of two or more star drifts, does not completely eliminate from the residuals of individual trapeziums those systematic motions which are automatically eliminated when equal and opposite areas are combined with equal weights, and hence the accordance when opposite zones are combined is greater than it would be if the distribution of residuals over the individual trapeziums were fortuitous.

As the solution gives only $u \sin \Omega$, it is necessary to assume a value for Ω, or to form new equations to determine such a value, if we desire to find u or $u \cos \Omega$. The most likely plane of rotation is that of the galaxy. If we assume that the rotation just found is in that plane, then, taking $\Omega = 280°$ and the inclination of the galactic plane to the equator as $= 62°$, the centennial rotation should be $+0''{\cdot}15 \operatorname{cosec} 280° \operatorname{cosec} 62° = -0''{\cdot}17$, almost exactly the quantity that Poincaré desired. I have not thought it necessary to form equations to determine the plane of rotation. This has been done by Dr. Charlier,* who deduced in 1913 from the proper motions of fourth and fifth magnitude stars in Boss's *Preliminary General Catalogue* that our system of co-ordinates is rotating on a plane differing within the probable error of the solution from the plane of the Milky Way. Using a method differing widely from that of this paper, he concluded that the node of the invariable plane of the solar system on the plane of the Milky Way has a motion amounting to $+0''{\cdot}35$ per century. Geometrically there is no difference between a rotation of the Milky Way on a fixed plane of the ecliptic and a rotation of a fixed plane of the ecliptic on the plane of the Milky Way, except that the rotation should have opposite signs according to the method of expression. Reduced to my notation, Dr. Charlier's value of $u \sin \Omega$ is $+0''{\cdot}34 \pm 0''{\cdot}12$ as compared with the value $+0''{\cdot}15 \pm 0''{\cdot}06$ found above. He does not solve separately for right ascension and declination nor for different zones. The difference is somewhat larger than the probable error of a difference as deduced from the separate probable errors. Since Dr. Charlier's stars are included in the stars used in the present solution, and in view of the much smaller probable error attaching to my result, I do not propose to amend the value found in this paper. This value implies a complete rotation of the universe in 740 million years.

The effect of the rotation term on precession can be expressed by leaving the polar motion as resulting from Boss's equations unchanged, and substituting $\alpha - 15''$ for α in the formulæ for precession in right ascension and declination.

Assuming as before that the rotation is on the plane of the galaxy we obtain for $u \cos \Omega$ the value $+0''{\cdot}15 \cot \Omega = +0''{\cdot}15 \cot 280° = -0''{\cdot}03$. Boss found from his equations,

$$\Delta k = -0''{\cdot}37 \pm ''{\cdot}06$$
$$\Delta n = +0''{\cdot}34 \pm ''{\cdot}06$$

* "The Motion of the Stars," *Meddelanden från Lunds Astronomiska Observatorium,* serie ii., No. 9, pp. 76–8.

together with solar motion terms with which the present inquiry is not concerned.

For these terms it will be necessary to substitute

$$\Delta k = -0''\cdot37 \pm ''\cdot06$$
$$\Delta n' = +0''\cdot34 \pm ''\cdot06$$
$$\Delta n = +0''\cdot34 \pm ''\cdot06 + 0''\cdot03 \pm ''\cdot01 = +0''\cdot37 \pm ''\cdot06.$$

We then have as the correction to Newcomb's precession on the ecliptic reduced to right ascension $+0''\cdot37 \cot \epsilon = +0''\cdot85 \pm ''\cdot14$, including $-1''\cdot94 \cos \epsilon$, *i.e.* $-1''\cdot78$, geodesic precession,* so that the total correction to Newcomb's lunisolar precession in right ascension is $+2''\cdot63$ per century. The rotation on the galactic plane should set up a term of $u \cot 62° = +0''\cdot15 \operatorname{cosec} 280° \cot 62° = -0''\cdot08 \pm ''\cdot03$ in right ascension. The precessions on the ecliptic and galactic plane account, therefore, for $+0''\cdot77 \pm ''\cdot14$ of Δk. A further correction arises from a revision of the masses of the planets, as affecting planetary precession. Using $\frac{1}{3,500,000}$ for the mass of Mercury, and $\frac{1}{408.358}$, as found by me for the mass of Venus, I find a total value of $-13''\cdot19$ instead of Newcomb's value $-13''\cdot42$ for planetary precession in right ascension at epoch 1850, or of $-12''\cdot72$ as compared with Newcomb's value $-12''\cdot95$, for Boss's epoch 1875. We thus have a total correction of $+0''\cdot77 \pm ''\cdot14 + 0''\cdot23 \pm ''\cdot09$ † $= +1''\cdot00 \pm ''\cdot17$ to Newcomb's value of precession in right ascension, arising from a combination of the individual precessions. But, as has been seen, Δk, the correction to the total precession in a century from the assumed equinox amounts to $-0''\cdot37 \pm ''\cdot06$. In order to compare these widely discrepant values it will be necessary to discuss the position and motion of the equinox.

It may be well before proceeding further to draw attention to the system on which the probable errors have been computed. I have used throughout the probable errors derived from the discordances of the individual trapeziums and half-sectors, separate residuals being computed in right ascension and declination for each trapezium and half-sector, though I have also stated the probable error derived from the errors of separate zones or from their discordances. Boss uses two systems of probable errors. On the one system the errors are derived as in this paper; on the other system the probable error attaching to a given solution is found from the discordances of the three solar motion terms, X, Y, Z, computed independently for seven zones of declination. The errors thus obtained are attributed to the precessional terms and solar motion terms alike, according to their solution weights. This method appears to me to be illegitimate. It is characteristic of the different solutions given by Boss that the proper motion terms Y and Z show greater discordances than the precessional terms Δk, Δn, and the solar motion term X, and I find that my term $u \sin \Omega$ also shows smaller

* See J. A. Schouten in *Koninklijke Akademie van Wetenschapen te Amsterdam*, Proceedings of the Section of Sciences, 21 (1), (1918), pp. 533–37, and A. D. Fokker, *ibid.*, 23 (1), (1920), pp. 729–38. I adopt Professor Eddington's value for the geodesic precession, *The Mathematical Theory of Relativity* (1924), p. 99.

† This is the probable error implied in the weight which Boss assigns to Newcomb's determination. I have not investigated the probable error independently.

discordances than Y or Z. It would appear, therefore, that on the second system of probable errors Boss has allowed the probable errors of the precessional terms to be affected by the discordances of different solutions for solar motion terms which do not affect the determination of the precessional terms. I owe to Professor Eddington the suggestion that the probable errors are to be regarded as errors of sampling, depending on the different degrees in which different star drifts have entered into the means of the different trapeziums.

A discussion similar to Boss's is given by Professor Eichelberger in *Astronomical Papers of the American Ephemeris*, vol. x, part i, "Positions and Proper Motions of 1504 Standard Stars for the Equinox 1925·0 (1925)," pp. 158, 159. Professor Eichelberger uses 1198 stars only as compared with Boss's 5413, and it may be presumed that all or nearly all his stars were also used in Boss's solution. He omits from consideration the two polar caps $\pm 80°$ to $\pm 90°$, and his results are affected by much larger probable errors than Boss's. I find also that the systematic differences between his proper motions in declination and Boss's do not affect the result. I have, therefore, thought it unnecessary to undertake the labour of introducing an additional unknown into his equations as well as into Boss's. I have to thank Professor Eichelberger for drawing my attention to a clerical error in his first normal equation in declination, where $+0''{\cdot}082$ should be $-0''{\cdot}084$. The equations were solved with the correct figures.

The discrepancy between the total precession from the assumed equinox and the sum of the individual precessions is just as marked in Professor Eichelberger's as in Boss's solution.

The Equinox Correction.

This correction is usually expressed in the form R.A. from True Equinox *minus* R.A. from Assumed Equinox.

Newcomb in his *Astronomical Constants* (1895) expresses it in the opposite sense, True Equinox *minus* Assumed Equinox. In the present paper the term will be reckoned in the usual direction. The correction to be applied to the equinox of Newcomb's system N_1 was closely examined by Newcomb in *Astronomical Constants* in the light of meridian observations of the Sun, Venus, and Mercury, and indirectly of transits of Venus and Mercury. The meridian observations discussed extended from 1750 to 1892. The correction there found amounted to $-0''{\cdot}48 + 0''{\cdot}30T$, where T is reckoned in centuries from 1850·0. Newcomb does not give the probable errors. If the probable error of the centennial term is computed from the discordances of the different solutions on which it is based, it should amount to $\pm''{\cdot}30$. This value of the centennial term was treated by Newcomb himself in his *Precessional Constant*,* by Hough and Halm in their paper "On the Systematic Motions of the Bradley Stars," *M.N.*, **70** (1910), 587, and by Boss in the discussions already cited, as the best value determined from the observations. Newcomb's discussion appears, however, to suffer from one grave defect.

* *Astronomical Papers of the American Ephemeris*, vol. viii. (1897).

Since the observations in right ascension give the position of the Sun as referred to the assumed equinox, and the observations in declination give his position as referred to the true equinox, the difference between the two observed places will correspond to the equinox correction if the observations compared are simultaneous, or at least if the uneliminated tabular errors are the same in both classes of observation, but not otherwise. Now instead of making separate determinations of the equinox correction for successive epochs Newcomb discussed the whole series of R.A. observations of each luminary separately from the whole series of declination observations. The observations in right ascension and declination respectively have not the same distribution over the whole range of years considered, nor are the weights assigned to the two classes of observation distributed alike. And, since the discussion does not allow for the possibility of the motion of any of these luminaries being affected either by a secular acceleration or by a fluctuation, although it is now practically certain that both exist, it would appear to be necessary to set the discussion aside and rely on work free from this objection. The subject was rediscussed by Newcomb in his posthumously published *Researches on the Motion of the Moon*, part ii,* in the light of occultations from 1672 to 1908. He there expresses the opinion that a determination from occultations is less liable to error than a determination from meridian observations of the Sun, the latter being affected by diurnal change of temperature on the instrument and the surroundings, and still more so by the personal equation; to which it might be added that variation of latitude enters with its full force into observations of declination. In Newcomb's opinion the result obtained in that discussion, $-0''\cdot15\pm''\cdot12 + (0''\cdot21\pm0''\cdot30)T$, where T is reckoned in centuries from 1850·0, was the most reliable value obtained up to the date of the discussion. It is not open to the criticism that I have expressed on Newcomb's earlier determination.

Since then a great improvement has been made in meridian observations, and the observatories at Greenwich, Washington, and the Cape of Good Hope have succeeded in obtaining a remarkably consistent series of observations. These give a large negative correction to the N_1 system. From the Cape † we have the following means :—

	1907–11.	1912–16.
From Sun	$-0''\cdot96$	$-0''\cdot80$
" Mercury	−0·72	−0·69
" Venus	−0·56	−0·60

Washington ‡ gives us—

	1911–18.
From Sun	$-0''\cdot80$

* *Astronomical Papers of the American Ephemeris*, vol. ix., part i. (1912), pp. 212–22.

† *Annals of the Cape Observatory*, vol. viii., part iv. (1915), 38 D; part v. (1921), 28 E.

‡ See Mr. Cullen, *M.N.*, **85** (1925), 1000.

From Greenwich * we have—

	1915–21.	1922–24.
From Sun	−1″·10	−0″·96
	1915–23.	
" Venus	−0″·95	

Older observations at these observatories are discordant from these and from each other. From the observations summarised above I deduce for 1916 a value of −0″·88±″·06.

An analysis of the occultations used by Newcomb shows that his weight 100 corresponds to a probable error of ±″·25. I therefore add to his nine groups of determinations from the occultations a group derived from the meridian observations with epoch 1916 and weight 1600.

The series of group-determinations now becomes:—

No.	Epoch.	Observed Correction.	Weight.	Residual.
		″		″
1	1720	+0·15	16	−0·53
2	1761	−4·05	4	−4·42
3	1792	−1·02	25	−1·15
4	1811	+0·06	16	+0·07
5	1829	+0·12	64	+0·27
6	1848	−0·03	100	+0·26
7	1865	+0·15	100	+0·57
8	1882	−0·30	100	+0·25
9	1899	−0·18	100	+0·50
10	1916	−0·88	1600	−0·07

From these I obtain by a least squares analysis

$$\text{Equinox correction} = -0''\cdot36 \pm ''\cdot12 - (0''\cdot76 \pm 0''\cdot23)\text{T}$$

where T is expressed in centuries reckoned from 1857·0. Reduced to 1925·0 this becomes −0″·88−0″·76T. Sir Frank Dyson in the *First Greenwich Catalogue* for 1925·0 finds from Greenwich meridian observations alone for epoch 1925·0 a value of −0″·90−0″·45T. This seems to me to assume too large and consistent an error in the nineteenth century in meridian observations and occultations alike. Mr. Cullen admits a correction of −0″·90 for epoch 1925·0, but casts doubt on the secular term on the ground that, if the recent observations had been affected by the same personality as those immediately preceding 1915, and had not been corrected for the annual term in variation of latitude, they would not have shown the large negative correction that is known to exist. These criticisms might apply to the Greenwich meridian observations, but it is difficult to see how a uniform personality or annual term in variation of latitude can have affected Newcomb's series of occultations.

* See *First Greenwich Catalogue of Stars* for 1925·0 (1924), viii., ix., xi., also articles in *M.N.*, by Dr. Jackson, **84** (1924), 476–81, and Mr. Cullen, *ubi supra.*

Professor Eichelberger * assumes that the equinox of Newcomb's system N_1 was correct for 1840, although Newcomb in his *Astronomical Constants* had wished to apply a correction of $-0''.51$ at that epoch, and in his later work had found a correction of $-0''.17$ for the same epoch. Professor Eichelberger also regards $-0''.60$ as the proper correction for 1910, for which I find $-0''.76$. From these two values he deduces the centennial term $-0''.86$.

It will be noticed that the residuals shown in the table given above appear to range, attaining a maximum about 1878. If this indicates a real fluctuation in the motion of the equinox, it may perhaps be best explained as the result of an oscillation of the plane of the Earth's orbit. I do not, however, propose to assume such a fluctuation in the present discussion.

Apparent Discrepancy in the Observations.

In an earlier part of this paper it was found that while the individual precessions gave a total correction of $+1''.00 \pm ''.17$ to Newcomb's value, the total precession from the assumed equinox was $0''.37 \pm ''.06$ less than Newcomb made it, leaving an unexplained precession of $-1''.37 \pm ''.18$ on the equator. We have just found that a correction of $-0''.76 \pm ''.23$ per century is required to reduce right ascensions from the assumed to the true equinox. We thus get a total unexplained precession on the equator of $-2''.13 \pm ''.29$ per century. As the value of this term amounts to seven times its probable error it is most unlikely that it should be an accidental error. It may be observed that a similar term arises from Professor Eichelberger's discussion. He finds for Δk reduced to the equinox of Newcomb's system N_1 a value of $-0''.65$ per century and for Δn $+0''.22$. From the latter he deduces $+0''.51$ for the correction to lunisolar precession. This leaves $-1''.16$ to be explained. If we apply, as above, $+0''.23$ as a correction to planetary precession, and $-0''.76$ as a correction to the equinox, the unexplained precession will amount to $-1''.16-0''.23-0''.76=-2''.15$, practically the same value that I have deduced from an analysis of Boss's equations.

This undiscovered term has long exercised a disturbing influence on attempts to determine precession. With the form of equation which as has been seen was used by Boss's predecessors, it affected the determination of precession from right ascensions, but not the determination from declinations. Thus Newcomb found for general precession in longitude from right ascensions $+0''.86$ per century more than from declinations.

Boss's normal equations give $+0''.05$ more from right ascensions than from declinations for polar motion and, therefore, $+0''.13$ more from right ascensions than from declinations for lunisolar precession in longitude, only one-seventh part of the discrepancy found by Newcomb. Newcomb was of the opinion that $+0''.30$ of the discrepancy consisted of an error in the equinox, but after much study he was unable to trace the cause of the remaining discrepancy, though he expressed the opinion that the main cause of discordance was an error of double period in the Bradley right ascensions. He finally adopted a value

* *Op. cit.*, p. 21.

exceeding by +0″·16 that derived from the right ascensions when corrected for the equinox, thus assuming that the value from declinations required a correction of −0″·40.

In 1910 Hough and Halm, accepting Newcomb's separate determination from declinations, proposed to explain the outstanding difference of 0″·56 by an unequal distribution of two star drifts. This could hardly affect Boss's determination of precession where the solution is based on a survey of the whole sky, equal areas receiving equal weights ; we have just seen that the particular discrepancy which it seeks to remedy no longer exists, and in any case 0″·56 in longitude or 0″·51 in right ascension is only the fourth part of the apparent discrepancy with which we have now to deal.

Boss's equations gave him the correction +1″·15±″·20, the $\Delta' m - \Delta k$ of his paper, which in his opinion could contain nothing except the equinox correction and the correction to planetary precession in right ascension. He considered Newcomb's determination of the centennial term in the equinox correction as given in *Astronomical Constants*, viz. +0″·30, to be the best, and he assigned to it a weight corresponding to a probable error of ±0″·25. He also assigned a weight corresponding to a probable error of ±0″·09 to the theoretical determination of planetary precession. With the weights appropriate to these probable errors, he obtained a further addition of +0″·49 to the centennial equinox correction, bringing it up to +0″·79; he also obtained a correction of −0″·06 to the planetary precession reckoned negatively, of −0″·25 to his own determination of the lunisolar precession in right ascension, and of +0″·05 to the value which he had obtained for mean motion in right ascension measured from the equinox of system N_1. Such a distribution of the discrepancy over the different terms is no longer possible. Instead of a correction of +0″·79 per century to the equinox we have found it necessary to make a correction of approximately equal value in the opposite direction, and no possible correction to the known precessions could account for more than a small part of the apparent discrepancy.

Motion of the Earth's Orbital Plane.

As the unexplained term exists in right ascension only, it can be explained only by a motion of the plane of the ecliptic in relation to a mean of the fixed stars. This might be explained as a motion of the " invariable " plane of the solar system, but it seems easier to regard it as a motion of the Earth's orbital plane due to some cause other than the known action of the planets. Such a motion would be geometrically indistinguishable from planetary precession. At present the theoretical and observed values of the motion of the node of Mercury differ by about $1\frac{1}{2}$ times the probable error of a difference, and those of the node of Venus differ by about $5\frac{1}{4}$ times the probable error of a difference, but there is good agreement between theory and observation in the case of the node of Mars. The motions of the poles of the orbit referred to the Earth's equator show agreement within the probable error in the cases of Mercury and Mars, but the discordances exceed the probable error in

the case of Venus and the Earth. It has just been found that the Earth's node, the equinox, has a motion differing from its theoretical motion by 7 times its probable error. The reality of the discordance is well supported by the similar discordance in the case of Venus, and the observations of Mercury do not tell against it. I therefore propose to introduce a new precession on the equator, amounting to $-2''\cdot13\pm''\cdot29$ per century, and I suggest as its explanation a motion of the plane of the Earth's orbit similar to that which has been observed in the case of Venus, but for which no gravitational explanation is as yet forthcoming.

Precessional Constants for 1925·0.

The following are the values of the centennial precessional motions for epoch 1925·0 deduced from this discussion.

Precession in right ascension:

	Newcomb's Value.	Correction.	Concluded Value.
	"	"	"
Lunisolar	+4621·20	+2·63	+4623·83
Geodesic		−1·78	− 1·78
Galactic		−0·08	− 0·08
Planetary	− 12·00	+0·23	− 11·77
Empirical precession on the equator		−2·13	− 2·13
Total	+4609·20	−1·13	+4608·07

Precession in longitude:

Lunisolar	+5037·21	+2·87	+5040·08
Geodesic		−1·94	− 1·94
Galactic		−0·09	− 0·09
Planetary	− 11·02	+0·22	− 10·80
Empirical precession on the equator		−1·95	− 1·95
Total	+5026·19	−0·89	+5025·30
Motion of the equatorial pole	+2004·47	+0·34	+2004·81

Precession in right ascension:

$$+307^{s}\cdot205+133^{s}\cdot654 \sin(\alpha-1^{s}\cdot0)\tan\delta.$$

Precession in declination:

$$+2004''\cdot81\cos(\alpha-15'').$$

Constant correction to the right ascensions of Boss's Catalogue:

$$-0^{s}\cdot059-0^{s}\cdot051T.$$

Constant correction to the right ascensions of Eichelberger's Catalogue:

$$-0^{s}\cdot010+0^{s}\cdot006T$$

where T is the time elapsed in tropical centuries since 1925·0.

Correction to annual proper motions in right ascension of Boss's Catalogue :

$$+0^s\cdot00025-0^s\cdot00023 \sin\alpha \tan\delta+0^s\cdot00010 \cos\alpha \tan\delta.$$

Correction to annual proper motions in right ascension of Eichelberger's Catalogue :

$$+0^s\cdot00082-0^s\cdot00023 \sin\alpha \tan\delta+0^s\cdot00010 \cos\alpha \tan\delta.$$

Correction to annual proper motions in declination of Boss's Catalogue :

$$-0''\cdot0034 \cos\alpha-0''\cdot0015 \sin\alpha.$$

Correction to annual proper motions in declination of Eichelberger's Catalogue :

$$-0''\cdot0034 \cos\alpha-0''\cdot0015 \sin\alpha.$$

University Observatory, Oxford :
1926 *March* 24.

Discussion of the Greenwich Observations of the Sun, 1836–1923. By H. Spencer Jones, M.A., Sc.D., H.M. Astronomer at the Cape.

1. A discussion by P. H. Cowell of the Greenwich observations of the Sun during the period 1864–1900, in which Leverrier's tables were used in the *Nautical Almanac*, was given in *M.N.*, *R.A.S.*, **66**, 302–307. For other purposes I have made a comparison between the Greenwich observations of the Sun and Newcomb's tables for the period 1836–1863, in which the *Nautical Almanac* positions were derived from Carlini's tables. It therefore seemed desirable to rediscuss the observations from 1836 up to 1923, the latest year for which the Greenwich results are available, using Newcomb's tables as a basis of comparison for the whole period, with a view to the derivation of corrections to Newcomb's adopted mean longitude, eccentricity, and perihelion, and to the mass of Venus, and with a view also to a discussion of the motion of the equinox.

2. *Correction for Equinox.*—Previous to 1849 the right ascensions of the stars contained in the Greenwich clock list were revised from year to year, the adopted places being given in the introductions to the several volumes of Greenwich observations. In 1849 the system was inaugurated of making the clock-star list depend upon a definite catalogue place, and from 1849 to 1902 the clock-star places were based upon the successive Greenwich catalogues. Since 1902, Newcomb's Catalogue has been used as a basis for the reductions.

A comparison of the Greenwich clock-star lists for the period 1836–1895 with the Greenwich Five-Year Catalogue, 1890, was given by W. G. Thackeray.* Auwers's proper-motions and Struve's precessions

* *M.N.*, *R.A.S.*, **56**, 466, 1896.

were used as a basis for the comparison. Corrections were derived in this way to reduce the adopted clock-star lists to the basis of the Five-Year Catalogue, 1890. For the period 1847–1902 the corrections necessary to reduce the several Greenwich catalogues, on which the clock-star lists subsequent to 1848 were based, to Newcomb's equinox are given by E. W. Brown * on the authority of Hedrick. Correcting Thackeray's comparison to the basis of Newcomb's precession and proper-motions, and adopting the values computed by Hedrick for the period 1849–1902, we obtain the following corrections to reduce the clock-star lists to Newcomb's equinox, and these have been adopted in the present discussion :—

TABLE I.

Table of Adopted Corrections to reduce the Greenwich Clock-Star Lists to Newcomb's Equinox.

Year.	Correction. s	Year.				Correction. s
1836	−·024	1847				+·079
37	−·016	48				+·085
38	−·017	49–55	depends on	Gwh.	'45	+·024
39	−·002	56–61	,,	,,	'50	+·008
40	·000	62–69	,,	,,	'60	+·024
41	−·008	70–77	,,	,,	'64	+·034
42	·000	78–88	,,	,,	'72	+·029
43	+·078	89–92	,,	,,	'80	+·037
44	+·086	93–98	,,	,,	5 yr. '90	+·049
45	+·093	99–02	,,	,,	10 yr. '90	+·054
46	+·065	1903–				·000

3. *The Mean Longitude, Eccentricity, and Perihelion.*—For the derivation of corrections to Newcomb's adopted mean longitude, eccentricity, and perihelion, the mean differences by months of the Sun's longitude (Tabular *minus* Observed) were used. The method adopted by Cowell was followed, in which each month receives equal weight in the analysis, irrespective of the number of observations. It is further assumed that for each monthly group-mean, the longitude of the Sun has its value at the middle of the month. These assumptions result in a great saving in numerical computation, and in dealing with a large mass of evenly distributed material only a slight increase in accidental error is involved.

For the periods 1836–1863 and 1901–1923 direct comparison with Newcomb's theory was available, the tabular positions for the earlier epoch having been computed from Newcomb's tables of the Sun. For the period 1864–1900 the volumes of Greenwich observations give a comparison with Leverrier's tables. For the present purpose it was sufficient to reduce Leverrier's tabular positions to Newcomb's only in so far as the terms depending on the Sun's mean longitude

* *M.N., R.A.S.*, **73**, 700, 1913.

are concerned. A comparison of the long-period perturbations in the longitude of the Sun, according to the two tables, revealed no differences sufficiently important to be taken into account.

Neglecting small terms in T^2, which are not of sufficient magnitude to be taken into account, the differences between the two tables in the sense Leverrier *minus* Newcomb are as follows :—

Mean longitude	+0″·68	+ 0″·82T
Perihelion	−7·8	−17·26T
Mean anomaly	+8·5	+18·08T
Principal elliptic coefficient in longitude	−0·52	− 0·27T,

where T denotes centuries from 1900·0.

The longitude difference, Leverrier *minus* Newcomb, may therefore be taken as

$$+0''\cdot68+0''\cdot82T-(0''\cdot52+0''\cdot27T)\sin(E-100^\circ)+(0''\cdot28+0''\cdot60T)\cos(E-100^\circ)$$
$$=+0''\cdot68+0''\cdot82T-(0''\cdot57+0''\cdot42T)\sin\theta+(0''\cdot14+0''\cdot51T)\cos\theta.$$

In these expressions, E denotes the Earth's longitude and θ an angle which has the value 0° for January, 30° for February, 60° for March, and so on.

If now we measure T from the mean epoch 1880·0, and denote T so measured by T′, we derive the following longitude differences, Leverrier *minus* Newcomb :—

Jan.	+0″·55	+1″·33T′	July	+0″·48	+0″·31T′
Feb.	+0·31	+1·05T′	Aug.	+0·72	+0·59T′
Mar.	+0·12	+0·71T′	Sept.	+0·91	+0·93T′
Apr.	+0·03	+0·40T′	Oct.	+1·00	+1·24T′
May	+0·08	+0·21T′	Nov.	+0·95	+1·43T′
June	+0·25	+0·17T′	Dec.	+0·79	+1·47T′.

The mean monthly differences (Leverrier *minus* Observed) for the period 1864–1900 were corrected by the above amounts to reduce to Newcomb. The effect of other short-period differences between the two tables can be neglected in an analysis for terms of annual period.

The observations for the whole period were corrected for equinox, using the corrections given in Table I.

Dealing with 4-yearly periods we derive the figures given in Table II., wherein each entry represents the sum of the differences in longitude (Newcomb *minus* Observed) for the four years concerned. For the sake of symmetry the errors for the months 1891 September and 1915 January to March have been supplied by interpolation. The observations from 1915 onwards have been corrected by −0ˢ·040 to allow for the change of personality which occurred when the moving-wire method of observation was substituted for the tapping method.*

* *Vide* R. T. Cullen, *M.N.*, *R.A.S.*, **85**, 1003, 1925.

TABLE II.

Mean Errors of Sun's Longitude (Newcomb–Observed).

Unit $0^s.01$.

Years.	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
1836–39	−17	− 1	−14	−34	−10	−26	−26	−21	−23	−22	− 3	+ 8
40–43	−14	−13	− 1	−14	−30	− 3	−18	− 8	− 4	− 1	+33	+17
44–47	+14	+10	− 4	− 6	− 3	−14	−11	−14	0	+10	+25	−10
48–51	− 5	−10	−13	+ 5	− 8	−23	−37	− 4	−18	−19	−31	− 5
52–55	+ 7	− 5	− 7	− 4	−16	−11	−23	−33	−17	−26	−10	+14
56–59	−20	− 3	−12	− 8	+ 7	− 9	−13	− 7	−27	−35	−13	− 7
60–63	+ 4	− 2	− 7	+ 7	+25	−15	− 2	− 8	− 4	− 5	− 6	+11
64–67	−36	−29	−21	− 9	−24	−28	−11	−19	−25	−23	−22	−14
68–71	−31	−37	−38	−20	−11	−15	− 9	−24	−11	−19	−10	−11
72–75	− 9	−12	−16	−24	− 4	+ 1	−23	−21	−12	−15	− 5	− 9
76–79	− 8	−14	−16	−13	− 4	0	− 1	−10	−25	− 4	− 1	− 8
80–83	− 1	− 5	−13	−12	− 1	+ 3	− 5	−13	−19	−18	− 4	0
84–87	+ 1	−17	− 7	+ 2	− 1	+11	−12	− 9	− 6	−14	+ 5	− 1
88–91	− 5	− 5	0	− 5	−11	− 1	−12	− 9	−14	−22	+ 8	− 5
92–95	+ 9	+ 3	− 4	+10	+10	+15	+14	− 1	+ 1	− 6	+ 8	+ 1
96–99	+ 6	− 1	− 1	+ 4	+16	+12	+ 9	+ 7	+ 2	+12	+ 4	+ 8
1900–03	+ 4	− 2	+ 5	+ 1	+11	+16	− 1	− 2	− 2	−15	+ 4	+ 2
04–07	− 7	− 8	− 8	− 5	− 3	− 8	−10	−17	− 6	−10	−10	−11
08–11	−18	−21	−20	−19	−19	−12	−22	−26	−44	−24	−29	−21
12–15	−27	−23	−25	−25	−12	−22	−16	−20	−27	−26	−31	−29
16–19	−34	−25	−21	−15	−23	−27	−29	−34	−28	−28	−27	−32
20–23	−22	−19	−14	−23	−27	−22	−39	−38	−31	−30	−29	−23

These residuals have been analysed by least squares with the following results :—

$$\Delta\odot = -0^s.0266 - 0^s.0230T' + [0^s.0043 + 0^s.0228T'] \sin\theta + [0^s.0048 - 0^s.0262T'] \cos\theta$$

$$\pm 0^s.0004 \pm 0^s.0016 \quad \pm 0^s.0006 \pm 0^s.0023 \pm 0^s.0006 \pm 0^s.0023$$

or

$$\Delta\odot = -0''.40 - 0''.34T' + [0''.08 + 0''.23T'] \sin(E - 100°) + [0''.05 - 0''.47T'] \cos(E - 100°)$$

$$\pm 0''.006 \pm 0''.025 \pm 0''.009 \pm 0''.035 \pm 0''.009 \pm 0''.035.$$

The correction required by Newcomb's mean longitude is therefore

$$+0''.40 \pm 0''.006 + (0''.34 \pm 0''.025)T',$$

T′ being measured in centuries from 1880.

This value may be compared with the following recent determinations :—

		Present Investigation.
From Cape observations of Sun—	″	″
Mercury and Venus, 1906–11 *	+0·29	+0·50
,, ,, 1912–16 †	+0·50	+0·52
From Greenwich observations of Venus, 1915–23 ‡	+0·47	+0·53

The agreement is sufficiently good, but should not be stressed for two reasons. In the first place, the present determination is not fundamental in the sense that the determinations with which it is compared are ; the Sun's declinations have not been used to derive an equinox correction. Instead, Newcomb's equinox has been adopted *ab initio*. Secondly, the mean longitude of the Sun is subject to irregular fluctuations, somewhat similar to, but smaller in amount than, the irregular fluctuations in the Moon's longitude. Thus, in the present solution, the residuals are predominantly positive for the period 1892–1903, and predominantly negative for the period 1908–1923. The correction to Newcomb's mean longitude here derived is that which brings theory and observation into the best general agreement over the period considered, but both the correction to the longitude at epoch and the correction to its mean motion would have had somewhat different values if the discussion had been concerned with the observations secured in a different term of years.

Referred to epoch 1900 as the origin for T, the correction to Newcomb's value of the Sun's mean longitude is

$$+0''\cdot47+0''\cdot34\mathrm{T}.$$

With this correction, both the longitude at epoch and the mean motion fall between the values adopted by Newcomb and Leverrier respectively.

The derived correction to the principal elliptic term in longitude is

$$-0''\cdot08\pm0''\cdot009-(0''\cdot23\pm0''\cdot035)\mathrm{T}.$$

The effect of the correction is again to make the derived value at epoch, and also the derived secular variation, fall between the values adopted by Newcomb and Leverrier. The value at epoch is nearer to Newcomb's value, but the secular variation is nearer to Leverrier's value.

Referred to epoch 1900 as the origin for T, the correction to Newcomb's value of the eccentricity is

$$-0''\cdot06-0''\cdot115\mathrm{T}.$$

From a discussion of the Cape observations of the Sun during the years 1907–1916, Hough § derived a correction to Newcomb of amount $-0''\cdot18$, with a probable error considerably larger than that here ob-

* *Cape Annals*, **8**, part iv., 1915.
† *Ibid.*, part v., 1921.
‡ *M.N., R.A.S.*, **84**, 479, 1924.
§ *Cape Annals*, **8**, part v., p. 37E.

tained. The two determinations agree in indicating a negative correction to Newcomb's eccentricity.

For the correction to perihelion we have

$$2e(\Delta\odot - \Delta\varpi) = -0''\cdot05 + 0''\cdot47\mathrm{T}',$$

and using the derived value of $\Delta\odot$,

$$-2e\Delta\varpi = -0''\cdot06 + 0''\cdot46\mathrm{T}$$

$$\text{or} \quad \Delta\varpi = +1''\cdot8 - 13''\cdot8\mathrm{T},$$
$$\pm 0\cdot3 \quad \pm 1\cdot1.$$

Referred to epoch 1900 as the origin for T, this becomes

$$\Delta\varpi = -1''\cdot0 - 13''\cdot8\mathrm{T}.$$

As in the case of the eccentricity, both the value at epoch and the secular variation are intermediate between the values of Newcomb and Leverrier, the value at epoch approximating to Newcomb's value, the secular variation approximating to Leverrier's value.

The mean correction derived from the two Cape series, 1907–1911, 1912–1916, was $-5''\cdot8$.* The present investigation gives, for the same epoch, $-2''\cdot5$. Both corrections are in the same sense, but, as in the case of the eccentricity, the two Cape series require a correction to Newcomb of larger amount than is here obtained.

The large correction to Newcomb's adopted mean motion of the perihelion here derived needs some comment. Newcomb's mean motion was based upon observations of the Sun, Mercury, and Venus, the observations of the Sun giving a value for $eD_t\delta\pi$ of $+0''\cdot32$, those of Mercury giving a value $-0''\cdot29$, and those of Venus $+0''\cdot46$, the relative weights being 5, 1, 1.† The finally adopted mean correction to Leverrier's value is therefore based to a large extent upon the solar observations, and an inspection of the table on p. 22 of Newcomb's *Astronomical Constants* shows that the large correction to Leverrier's mean motion which Newcomb derives is due entirely to the early Greenwich observations. The determination of the correction to the mean motion of the perihelion is peculiarly liable to systematic error; whereas the correction to the eccentricity depends mainly on observations secured near the two equinoxes and therefore under similar conditions, the correction to the perihelion depends mainly on observations secured near the solstices and therefore under widely differing conditions of temperature, etc., which may give rise to errors of a systematic nature. Newcomb recognised that the perihelion corrections given by the early Greenwich observations were abnormal,‡ and therefore gave them a gradually diminishing weight from 1750 to 1786, rejected them from 1786 to 1818, and gave a reduced weight from 1819 to 1834. Leverrier,§ on the other hand, concluded that the anomalies shown by the Greenwich results for the whole period 1750–1815

* *Cape Annals*, **8**, part v., p. 37E.
† *Astronomical Constants*, p. 88.
‡ *Ibid.*, pp. 27–28.
§ *Annales de l'Observatoire de Paris*, t. iv., p. 91.

were due to instrumental causes and not to a real motion of the perihelion.

If the perihelion corrections as derived by Newcomb for the period 1815–1892 from the Greenwich observations, and given on p. 22 of *Astronomical Constants*, are analysed, the derived centennial correction to the motion of the perihelion ($eD_t\delta\pi''$) is $-0''\cdot03$. Newcomb, including earlier observations with reduced weight, as stated above, obtained the value $+0''\cdot49$. From all other modern observations (excluding Paris) he obtained $+0''\cdot05$, and from all solar observations combined $+0''\cdot32$. His adopted value, in which the planetary observations have a small weight, was $+0''\cdot26$.

The present discussion, which excludes all observations prior to 1836, and which includes 31 years' observations later than those discussed by Newcomb, gives a value for the motion of the perihelion approximating to Leverrier's value, who—as we have seen—discarded the early Greenwich observations entirely. The observations from 1893 to 1923 therefore agree with those from 1836–1892 in indicating that only a very small correction to Leverrier's adopted mean motion of the perihelion is necessary.

The early Greenwich observations appear to be partly responsible also for the difference in the secular variation of the eccentricity here derived and that adopted by Newcomb. The separate values of the correction derived by Newcomb for periods of about 4 years each from 1750 to 1834 have a range of $0''\cdot64$, whereas those from 1835 to 1892 are remarkably uniform with a range of only $0''\cdot18$. The correction to the secular variation derived from the latter period alone is $-0''\cdot08$, whereas the inclusion of the earlier values results in a correction of $+0''\cdot22$, though this value is supported by the other solar observations discussed and also by the observations of Mercury and Venus. Nevertheless, the more modern Greenwich observations indicate that this value is too large, and, as seen above, agree with recent Cape observations in giving a negative correction to Newcomb's eccentricity.

Applying the corrections derived above to Newcomb's values for the mean longitude, eccentricity, and perihelion, the values resulting from the present investigation are as follows :—

$$\odot = 279^\circ\ 41'\ 48''\cdot51 + 129\,602\,768''\cdot47\mathrm{T} + 1''\cdot089\mathrm{T}^2$$
$$e = 3455''\cdot09 - 8''\cdot736\mathrm{T} - 0''\cdot0260\mathrm{T}^2$$
$$\varpi = 281^\circ\ 13'\ 14''\cdot0 + 6175''\cdot23\mathrm{T} + 1''\cdot63\mathrm{T}^2 + 0''\cdot012\mathrm{T}^3.$$

3. *The Mass of Venus.*—For deriving a correction to Newcomb's adopted mass of Venus, the convenient method used by Cowell was followed. This is based on the fact that the synodic period of Venus approximates to nineteen Julian months. If, therefore, the difference in longitude, Venus *minus* Earth, is represented in the form

$$\mathrm{V}-\mathrm{E} = 180^\circ + p \times 360^\circ/19,$$

p will increase approximately by unity in each month. To allow for the slight excess of the synodic period of Venus over nineteen Julian months, at intervals of 8 years 10 months, the same value of p is

assigned to two consecutive months. Thus $p=5$ is assigned to both 1885 September and October, $p=15$ to 1894 July and August, $p=6$ to 1903 May and June, etc.

The monthly mean differences (Newcomb *minus* Observed) are tabulated under the nineteen different values of p and the sums taken. But before doing this it is necessary to correct the residuals for the period 1864–1900, during which the *Nautical Almanac* places were derived from Leverrier's tables, so as to allow for the difference in the mass of Venus adopted by Newcomb and Leverrier respectively.

The principal Venus terms, as given by Newcomb, are

$$-4''\cdot838 \sin (V-E)+5''\cdot526 \sin 2(V-E),$$

and, as given by Leverrier, are

$$-4''\cdot933 \sin (V-E)+5''\cdot636 \sin 2(V-E).$$

These are based in the case of Newcomb upon a mass of Venus of 1/408000, and in the case of Leverrier upon a mass of 1/400246. Leverrier's terms when reduced to Newcomb's mass are practically identical with Newcomb's.

A correction,

$$+0''\cdot095 \sin (V-E)-0''\cdot110 \sin 2(V-E),$$

in the sense Newcomb *minus* Leverrier, was therefore used during the period 1864–1900 to reduce the whole material to the basis of Newcomb's adopted mass of Venus.

Corrections were also applied to reduce to Newcomb's equinox throughout, but corrections to allow for the difference between Leverrier and Newcomb in other short-period terms were not applied. Over the period 1864–1900, the mean effect of these on the terms in (V–E) will be negligible.

The following table gives the results obtained by summing for the various values of p :—

Table III.

Analysis of Terms in (V–E).

V–E.	p.	Sum of Errors (Tab.-Obs.). (Unit $0^s\cdot01$.)	No.	Mean. (Unit $0^s\cdot01$.)
198°·9	1	−127	55	−2·31
217·9	2	− 99	55	−1·80
236·8	3	−154	57	−2·70
255·8	4	−130	56	−2·32
274·7	5	− 69	56	−1·23
293·7	6	− 98	56	−1·75
312·6	7	− 74	56	−1·32
331·6	8	− 68	55	−1·24

TABLE III.—*continued.*

V-E.	*p.*	Sum of Errors (Tab.-Obs.). (Unit $0^s.01$.)	No.	Mean. (Unit $0^s.01$.)
°				
350°.5	9	− 72	55	−1.31
9.5	10	−107	55	−1.95
28.4	11	− 53	55	−0.96
47.4	12	− 87	56	−1.55
66.3	13	− 86	56	−1.54
85.3	14	− 30	56	−0.54
104.2	15	− 70	56	−1.25
123.2	16	− 79	56	−1.41
142.1	17	− 96	55	−1.75
161.1	18	− 87	55	−1.58
180.0	19	− 61	55	−1.11

Analysing the mean residuals in the form

$$a+b \sin (V-E)+c \sin 2(V-E)$$

and converting from time to arc, we derive the following values for the coefficients of the sine terms :—

$$b=+0''.049\pm0''.014$$
$$c=-0''.034\pm0''.014.$$

In other words, the correction to the Venus terms adopted by Newcomb, viz.,

$$-4''.838 \sin (V-E)+5''.526 \sin 2(V-E)$$

is

$$-0''.049 \sin (V-E)+0''.034 \sin 2(V-E)$$
$$\pm0''.014 \qquad \pm0''.014.$$

From each term separately, a correction to Newcomb's adopted mass of Venus can be derived. The term in sin (V−E) requires an increase in the mass in the ratio

$$1+\frac{0.049}{4.838} \text{ to } 1 \quad \text{or} \quad 1.0101\pm0.0029 \text{ to } 1.$$

Similarly the term in sin 2(V−E) requires an increase in the ratio

$$1+\frac{0.034}{5.526} \text{ to } 1 \quad \text{or} \quad 1.0061\pm0.0025 \text{ to } 1.$$

Both terms require an increase in the mass. From the weighted mean we find

$$\Delta M/M=+0.0080\pm0.0019,$$

corresponding to a value for the reciprocal of the mass of Venus of

$$404{,}700 \pm 800.$$

From the present discussion, therefore, the necessity for a slight increase in Newcomb's mass of Venus is indicated. The two principal terms in the perturbations of the Earth by Venus agree in requiring an increase. Newcomb's adopted mass is based to a large extent on the Greenwich observations of the Sun from 1750 to 1892; the period from 1750 to 1830, divided into sub-periods of 4 or 5 years, gave a somewhat large negative value for $\Delta M/M$ (the provisional value of M being 1/401847), whereas the period from 1830 to 1892 gave a very small correction.* The additional 30 years' observations (1893–1923) included in the present discussion also agree in requiring only a small correction to Newcomb's provisional value. The whole period, 1836–1923, indicates a mass for Venus which is in better agreement with Newcomb's provisional value than with his final value. Thus, while Newcomb's adopted correction to his provisional value is † $\Delta M/M = -0{\cdot}0119 \pm 0{\cdot}0028$ (m.e.), we have derived a correction to his final value of amount $\Delta M/M = +0{\cdot}0080 \pm 0{\cdot}0019$ (p.e.).

The discussion of the Cape Heliometer observations of Mars ‡ for the period 1899–1924 indicated, on the other hand, a correction to Newcomb's mass of Venus in the other direction, viz., $\Delta M/M = -0{\cdot}0115 \pm 0{\cdot}0048$, but the period covered by the observations is not of sufficient length to derive a correction with a small probable error.

The correction here derived is in close agreement with that obtained by Ross § from the discussion of all available meridian observations of Mars ($\Delta M/M = +0{\cdot}0112$).

4. *The Motion of the Equinox.*—For the derivation of the motion of the equinox, it is necessary to determine the corrections to the Sun's absolute right ascension from the declination observations. The Greenwich declination observations during the period under consideration have been based upon several different values for the refraction, different values for flexure, and different values and formulæ for the R–D discordance. These differences in the reductions for the various years may be summed up by saying that the differences in reduction from year to year appear in the monthly means "Observed *minus* Tabular declination" in the form $f(z)$, where z is the mean zenith distance of the Sun for each month, and $f(z)$ may have the form $\sin z$, $\tan z$, or $\sin z \cos^2 z$.

Now a difference $\Delta\delta = f(z)$ where $f(z)$ may have any of the forms specified, has its mean values at the equinoxes and its extreme values at the solstices. Hence when the monthly mean values are analysed by assuming the representation

$$\Delta\delta = a \cos \alpha + b \sin \alpha + c,$$

* Newcomb, *The Elements of the Four Inner Planets and the Fundamental Constants of Astronomy*, p. 22.

† *Ibid.*, p. 101.

‡ *M.N., R.A.S.*, **85**, 853, 1925.

§ *Astronomical Papers of the American Ephemeris*, **9**, part ii.

α being the right ascension of the Sun, the value derived for the coefficient a will be very small, since cos α has its extreme values at the equinoxes and its mean value at the solstices. In so far, therefore, as the correction to the Sun's absolute R.A. is concerned, no account need be taken of changes from year to year in refraction, flexure, or R–D discordance.

The effect of latitude variation corrections, on the other hand, needs to be taken into account, as was pointed out by W. G. Thackeray.* The 14-monthly component will not produce any systematic effect on the derived position of the equinox when averaged over a period of years; with the yearly period it is otherwise on account of the approximate constancy of phase of this component. The yearly term at any place is approximately of the form

$$A \cos (\theta + 120^\circ),$$

where θ=fraction of year$\times 360^\circ$. We may therefore with sufficient accuracy write $\theta - \alpha = 90^\circ$, and the yearly term is approximately

$$A \cos (\alpha + 210^\circ).$$

In analysing $\Delta\delta$, the yearly term in the latitude variation therefore affects principally the cos α term, on which the position of the equinox depends. Since A is of the order of one-tenth of a second of arc, the average effect on the position of the equinox will be of the order of $0''{\cdot}09$. Latitude variation has been applied in the reduction of the Greenwich solar observations in the annual volumes of Greenwich observations from 1902 onwards; for the discussion of the motion of the equinox it is best to remove the latitude variation corrections applied during this period in order to eliminate the systematic effect produced by the yearly term. This has been done recently by Cullen,† who has analysed the corrections applied and derived the resulting effect on the position of the equinox.

Up to 1914, the Greenwich observations of the Sun were analysed by deriving the mean monthly errors in ecliptic N.P.D. of the Sun's tabular place and analysing the mean monthly values in the form

$$\Delta\beta = a \cos \lambda + b \sin \lambda + c.$$

It can be easily shown that

$$a = \sin \epsilon (\Delta\lambda - \Delta\alpha),$$

where $\Delta\lambda$ is the error in the Sun's absolute mean longitude, as derived from its declinations, and is equivalent to Newcomb's $\Delta\alpha_\odot$, whilst $\Delta\alpha$ is the mean error in right ascension, and is equivalent to Newcomb's $\Delta'\alpha_\odot$. Thus if E is the equinox correction, the coefficient of cos λ in $\Delta\beta$ is E sin ϵ, so that E is derived directly. To the value of E so derived, a correction to reduce the adopted clock-star list to Newcomb's equinox must be applied.

* *M.N., R.A.S.*, **58**, 280, 1898.
† *Ibid.*, **85**, 1004, 1925.

TABLE IV.

Correction to Newcomb's Equinox.

Period.	Correction to Clock-Star List.	Reduction to Newcomb's Equinox.	Correction to Newcomb's Equinox.
	s	s	s
1836–40	−·095	−·012	−·083
41–45	+·023	+·050	−·027
46–50	+·038	+·055	−·017
51–55	+·017	+·024	−·007
56–60	−·017	+·008	−·025
61–65	−·001	+·021	−·022
66–70	−·024	+·026	−·050
71–75	+·014	+·034	−·020
76–80	+·015	+·032	−·017
81–85	+·015	+·029	−·014
86–90	+·048	+·032	+·016
91–95	+·057	+·044	+·013
96–00	+·051	+·051	·000
1901–05	+·013	+·022	−·009
06–14	−·038	·000	−·038
15–21	..	..	−·013
22–24	..	..	−·004

In the above table the effect of the corrections for latitude variation applied in the annual volumes of Greenwich observations from 1902 onwards has been eliminated. The figures for the period from 1915 to 1924 have been taken from Cullen's discussion of recent Greenwich equinox determinations; * they include a correction for change of personality consequent upon the introduction of the impersonal micrometer, and corrections to eliminate the effect of the latitude variation applied in the reductions.

The figures in the last column of Table IV. are liable to be affected by differences in personality in observing the Sun's limb and the stars, by variation with time of the personality of the mean observer, and by changes in the method of observing. The well-determined change of personality of amount $0^s.04$, which occurred when the impersonal micrometer was introduced in 1915, has been allowed for. No marked change of personality can be noticed in 1854, when chronographic registration superseded the eye and ear method. At the Cape, on the other hand, this change was accompanied by a large discontinuity in the derived equinox correction.†

Accepting the figures in the last column of Table IV. as derived in as uniform a manner as possible, and assuming that residual errors due to personality can legitimately be treated as accidental, we can analyse the figures in the form

$$E = x + yt.$$

The result of such an analysis, in which the correction for 1836-1840,

* *M.N., R.A.S.*, **85**, 1000–1006, 1925. † *Cape Annals*, **2**, part v., p. 10D.

which is markedly discordant, is bracketed, and in which equal weight is given to each year's observations, is

$$E = -0^s.016 + 0^s.014t \pm 0.003 \pm 0.012,$$

t being measured from 1883·5.

The derived correction to the motion of the equinox, as adopted by Newcomb, is therefore $+0^s.014$ per century, but the probable error attaching to the correction is of the same order as that of the correction itself. In view of the uncertainty resulting from variations in personality, we may conclude that the correction required by Newcomb's adopted motion of the equinox is very small, of the order of one-hundredth of a second of time per century. A longer period of observations, during which personality is controlled by the use of the impersonal micrometer, will be necessary to determine the correction with precision.

Eichelberger's recent determination * of a correction of $-0^s.057$ per century cannot be justified. It was derived on the assumption that Newcomb's equinox was correct for the mean epoch of its determination, about 1840, and that the mean correction to the right ascensions of Boss's Catalogue in the neighbourhood of the equator, as derived by Eichelberger, is due entirely to error in Newcomb's adopted motion of the equinox. A correction to Boss of $-0^s.04$ is therefore attributed to 70-years' error in the motion of the equinox. Apart from the doubtful legitimacy of the main assumption, an error of only $-0^s.02$ in the position of the equinox for 1840 (as is indicated above) would reduce Eichelberger's correction to the motion by one-half.

5. *Summary* (*a*).—The discussion of the Greenwich observations of the Sun made during the period 1836-1923 indicates that the following corrections are required by Newcomb's elements :—

Mean longitude	$+0''.47 + 0''.34T$
Eccentricity	$-0.06 - 0.115T$
Perihelion	$-1.0 - 13.8T$,

where T denotes centuries from 1900·0. The derived values at epoch and variations all fall between Newcomb's and Leverrier's values.

(*b*) The observed mean longitude is not represented within the limits of observational errors by theory. There are residual systematic fluctuations which, during the period under discussion, have an extreme amplitude somewhat exceeding one second of arc.

(*c*) A correction to Newcomb's mass of Venus is derived of amount

$$\Delta M/M = +0.0080 \pm 0.0019,$$

corresponding to a mass of 1/404,700. This is in close agreement with the mass derived by Ross from observations of Mars.

(*d*) A discussion of the motion of the equinox leads to the conclusion that the correction required by Newcomb's adopted motion is very small. A longer period of observations, during which personality is controlled by the use of the impersonal micrometer, will be necessary to determine the correction with precision.

* *Astronomical Papers of the American Ephemeris*, **10**, part i., p. 21.

Third Note on W. Herschel's Diffused Nebulosities.

By J. G. Hagen, S.J., Assoc.R.A.S.

1. Six more of W. Herschel's diffused nebulosities have been examined by the writer and by the Research Assistant, Dr. F. Becker. Convenient guiding stars were identified on the charts printed from Isaac Roberts' original negatives which, as is known, measure $2° \times 2°$. The guiding stars and their positions $\Delta\alpha$ and $\Delta\delta$ relative to the centres of Herschel's fields are listed in the following table. After the star is found on the chart, the $\Delta\alpha$ and $\Delta\delta$ have to be applied to it with opposite signs to obtain the place indicated by Herschel.

TABLE I.

H.'s No.	Centre (1925).	Guiding Star (1925).		Mag.	$\Delta\alpha$.	$\Delta\delta$.
	h m ° ′		h m ° ′		′	′
13	4 3·7+25 15	Camb. E. 1995	4 2·2+25 31	8·2	−21	+16
14	25·6+35 11	Lund 2254	24·3+35 6	6·8	−16	− 5
15	26·6+35 11	Lund 2254	24·3+35 6	6·8	−28	− 5
16	27·7− 7 27	Wien-O. 1150	30·2− 6 54	5·8	+38	+33
17	30·5+20 53	Berlin B. 1476	33·8+20 32	6·0	+45	−21
18	4 45·6+20 53	Berlin B. 1531	4 44·3+21 12	7·3	−18	+19

The description that Herschel gives of these nebulosities and his estimates of their extension in the parallel and in the meridian (here counted from the centre) are taken from the *Scientific Papers*, **2**, 463, and shown in Table II., together with the galactic latitude.

TABLE II.

H.'s No.	Parallel.	Meridian.	Description.	Gal. Lat.
	′	′		°
13	15	74	Much affected [with nebulosity]	−19
14	62	79	Suspected pretty strong nebulosity	− 9
15	57	79	Suspected nebulosity	− 9
16	15	68	Strong milky nebulosity	−34
17	15	78	Much affected	−18
18	15	78	Much affected	−15

The repetition of the number 15 in the second column may be explained by the fact that Herschel's field of view was 15′ in diameter, and that, as a general rule, he "allowed but a single minute of time for the extent of the nebulosity" in the direction of the parallel.

2. In what follows our own observations are stated, as estimated on the scale of five grades: I=thin or light, V=dense or strong. The description that Roberts has published of his plates is not repeated in any of these Notes.

Herschel No. 13.—The centre of this nebulosity has density V. The same density prevails in directions *sp* and *sf*, while the *np* portion of the field (surrounding the guiding star) and the *nf* portion are gradually fading towards IV and III.

Nos. 14 and 15.—From Table I. it appears that the two fields overlap in R.A., the two centres being only 16′ apart. The strip of Decl. common to both has density IV–V towards south and V towards north. The general field varies but little between densities IV and V. The dense cloud surrounding the guiding star is described in our *Preparatory Catalogue* (Specola Vat. X.), p. 132, because the nebula N.G.C. 1579 is only $6^{o8}=12'$ following the star.

No. 16.—The central field shows density V to the extent of 20′ or 25′ from the middle and is surrounded by densities IV, except in the four corners. Portions *np*, *nf*, and *sf* have III–IV and *sp* has V.

Near the guiding star there are two stars that deserve attention. The one *sf* has the photographic mag. 7, and the other *np* the mag. $7\frac{1}{2}$, while visually they are both below mag. 10. Neither is in the Bonn DM. Their positions relative to the guiding star are these:

Photographic mag. 7 : $\Delta\alpha=+7^{s}, \Delta\delta=-11'{\cdot}6$
„ „ $7\frac{1}{2}$: $=-31, \quad =+38'{\cdot}7$

No. 17.—The quadrant *sp* the centre of the field is covered with nebulosity V, the other three quadrants with densities IV–V.

No. 18.—The density of this field varies uniformly between grades IV and V, reaching V mainly in a *sp* direction.

3. We add two remarks of a general character:

In all the fields hitherto examined the nebulosities extend beyond the limits of the charts, *i.e.* beyond one degree from the centres.

Field No. 14 presents a sample of the difference between nebulæ that Herschel calls real and nebulosities that he calls milky. The nomenclature *Nebulæ* and *Nebulosities* for the two distinct classes of objects is perhaps the simplest that can be devised.

Vatican Observatory:
1926 *March.*

Melbourne Observatory—Comet Observations.

(*Communicated by the Government Astronomer for Victoria.*)

The following comet positions have been deduced from plates taken with the Melbourne Astrographic Telescope, the exposure (except for one plate) and measurement being carried out by Mr. W. M. Holmes. Two (in one case three) exposures were made on each plate, and the two (or three) positions deduced from a single plate have been combined, the mean result only being given in the appended lists. The positions are given in conformity with the resolutions of the 1925 Assembly of the International Astronomical Union.

Comet Reid 1925*b*.

	U.T.	Mean Place 1925. R.A.	Dec.	$\log P_\alpha$.	$\log P_\delta$.	Catalogue.
1925		h m s	° ′ ″			
May	30·53323	12 8 38·44	−35 25 9·5	$\bar{1}$·583	0·143*n*	Perth, 1900
June	12·37047	12 7 20·52	−37 50 22·1	$\bar{2}$·532*n*	$\bar{2}$·354	,, ,,
June	22·37758	12 12 38·81	−39 52 50·6	$\bar{2}$·913	$\bar{1}$·480	,, ,,
Aug.	7·47100	13 54 18·59	−50 38 52·8	$\bar{1}$·808	$\bar{1}$·919*n*	CpAS
Sept.	19·43362	17 12 2·31	−52 20 44·7	$\bar{1}$·696	$\bar{1}$·947	GZ
Sept.	25·47594	17 39 57·49	−51 13 20·8	$\bar{1}$·784	$\bar{1}$·410*n*	CpAS

Each position is derived from two exposures on the same plate.

Comet Ensor 1925.

	U.T.	Mean Place 1926. R.A.	Dec.	$\log P_\alpha$.	$\log P_\delta$.	Catalogue.
1925		h m s	° ′ ″			
Dec.	20·54342	2 18 13·04	−62 29 17·5	$\bar{1}$·745	0·425	CGA
1926						
Jan.	5·55967	23 31 54·69	−53 30 54·3	$\bar{1}$·888	0·597*n*	CGA
Jan.	12·87534	22 53 57·10	−47 44 8·6	$\bar{1}$·831	0·418*n*	Cape Zones, 1900
Jan.	23·47277	22 11 5·64	−38 31 35·6	$\bar{1}$·759	0·723*n*	Perth, 1900

Each of the first three positions is derived from two exposures on the same plate; the last position is derived from three positions on one plate.

Erratum.

Union Observatory, Johannesburg.

Error of Moon's Mean Longitude from Occultations.

The results from occultations are given on p. 209, but the "Time-Error" for 1923·6 *should read* 13·9 secs. and *not* 16·9 secs.

If recent occultations made here mainly at the time of 1st quarter are included, the table covering the use of Brown's Tables in the N.A. is :

	Time Error.		Error of Moon's Mean Longitude.	
	s		″	
1923·4	+14·6		+8·0	Includes observations made elsewhere : see U.O. Circ. No. 63.
23·6	+13·9		+7·6	
23·9	+13·6		+7·5	
24·1	+14·9		+8·2	
24·4	+16·3	(well-marked maximum)	+9·0	See U.O. Circ. No. 64.
25·4	+14·4		+7·9	,, ,, ,, 67.
25·6	+13·1		+7·2	
25·8	+13·0		+7·1	
1926·2	+13·2		+7·3	Unpublished.

Union Observatory :
1926 *April* 22.